Minimizing measurement uncertainty in calibration and use of accelerometers
ABSTRACT
This paper summarizes techniques and equipment used to minimize measurement uncertainty in the calibration of accelerometers. Background is given on the physics of vibration and design of accelerometers. Although referring primarily to comparison calibrations using sinusoidal vibration, the topics of shock and absolute calibration methods are included. Physics being universal, the topics should be just as pertinent to accelerometer use in the field as they are in the well-controlled laboratory.

INTRODUCTION
The paper was written to inform the reader about acceleration, accelerometers and their calibration, and to instill a healthy respect of the challenge of making reliable and accurate measurements (with any instrument, not just acceleration transducers). Mistakes are easy to make, and can be costly. With study and understanding, expensive mistakes might be avoided, and usually the biggest uncertainties are minimized.

There are three sections: First is a short summary of the "Do’s and Don’t’s" in accelerometer calibration. Then, a more lengthy second section discusses the physics of acceleration, accelerometers and calibration techniques. The final section describes the sources of uncertainty summarized in the first section, the reasons behind the "Do’s and Don’t’s," and techniques for estimation of uncertainty.

This is not a comprehensive listing of all the sources of uncertainty possible, nor of all the techniques useful for field application. More thorough coverage can come from many sources [1-5]. Instead the paper is intended to be brief and practical on each of the many topics. The difficulties are that the number of parameters in shock and vibration measurements is very large, and many of the parameters interact. Since most parameters go nonlinear at some extreme, the results can be chaotic, with wildly varying outcomes caused by seemingly minor input variations.

Two themes should stand out concerning measurement uncertainty. First, is quality: The best results are obtained with accelerometers and signal conditioning that have the best features for the application. Second is technique: Lack of good methods, understanding, or poor control of the pertinent parameters can result in poor results from even the best instruments. Another point is made on calibration accuracy: The overall uncertainty of an accelerometer used in the field can be no better than the uncertainty of its calibration in the laboratory. Overall uncertainty combines the uncertainties of all parameters, including those of the calibration.

A note of caution is advised when applying the results of a calibration, good or otherwise: The quality of a transducer should not be judged from a limited calibration, but rather from a thorough characterization. In a high quality calibration, numerous subtle performance parameters are controlled to establish the probable value of perhaps just one parameter, such as acceleration sensitivity. "Good" calibrations are possible on otherwise lousy accelerometers, since the controlled laboratory conditions could mask the characteristics that might cause problems in the field. On the other hand, an excellent transducer may seem to have poor performance with sloppy calibration. The following is a summary of recommendations for good calibration technique:
# DO'S AND DON'TS OF ACCELEROMETER CALIBRATION

| 1. General mounting       | Keep surfaces flat and clean  
|                         | Use recommended mounting torque (don’t guess; use a calibrated torque wrench)  
|                         | Assure full contact on transducer mounting surface  
|                         | Use couplant, such as oil (Improves response at high frequencies, and prevents surface damage during mounting)  
|                         | Mount for calibration as it will be used in the application, if possible (examples, actual cable and connectors, insulating mounting studs, adapting fixtures, etc.)  
|                         | Don’t nick or gouge surfaces  
|                         | Don’t overtorque (may damage shaker or transducer)  
|                         | Don’t undertorque (results in poor frequency response, or possible damage during shock)  
|                         | Don’t use more mounting interfaces than necessary  
|                         | Don’t use fixtures or cables which are larger or more massive than acceleration source can handle to provide good quality input  
| 2. Mounting with studs   | Use recommended mounting studs (with shoulder flange and correct thread length to prevent “bottoming out” in transducer)  
|                         | Don’t use too large a flange (it must fit completely in recess, otherwise transducer may be seated only on rim of shoulder)  
| 3. Mounting with adhesives | Use recommended adhesives, such as cyanoacrylate instant-setting  
|                         | Use appropriate solvents and procedures to remove adhesives (Note: many solvents may cause cancer)  
|                         | Don’t remove adhesively mounted units with the shock of a hammer blow or by twisting them off  
|                         | Never use razor blade or sandpaper to remove adhesives  
| 4. Fixtures              | Keep fixtures stiff and light, balancing trade-offs of amplitude and acceleration quality (due to maximum mass the exciter can take), with the required frequency response. Beryllium is best but is expensive, and particles are toxic if inhaled  
|                         | Design fixtures so mounting surfaces can be easily lapped  
|                         | Don’t orient the inertial center of mass of accelerometer (or overall center of gravity of fixtures) off the centerline of motion  
| 5. Cables                | “Finger tighten” only. (Exception: Sonic miniature connectors have hex flats. Use only supplied tools and recommended torque)  
|                         | Use only “noise treated” cable for high impedance piezoelectric accelerometers (unless they have low impedance output internal electronics)  
|                         | Connections in cabling must have complete and continuous shielding  
|                         | Use same cable or clamping technique during calibration as used in application, if possible  
|                         | Don’t use tools to tighten connectors. Don’t overtighten  
|                         | Don’t kink or knot cables, or overtighten cable clamps (insulation or noise treatment can be damaged)  
|                         | Don’t attempt to splice or repair noise-treated cable  
|                         | Don’t use damaged cables (even if damage is only suspected)  


## DO'S AND DON'TS OF ACCELEROMETER CALIBRATION

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| **6. Cable strain relief** | Clamp the cable if possible on the same structure the transducer is mounted  
Use free cable loop length 10 to 20 times cable diameter between transducer connector and first clamp  
Make sure clamp is at least 5 times wider than cable, to prevent tight bend radii and motion of inner conductors | Don’t tie down cable rigidly with a short, straight path, or with any preload. There should be no axial forces  
Don’t subject cable or connector to forces caused by relative motion between the transducer and the stationary end. Some transducers respond to cable-induced forces  
Don’t use long loops. Whipping may cause cable noise, forces and fatigue |
|   |   |   |
| **7. Grounding** | Prevent ground loops: Ground signal conditioning at one point only. (Example, if using Endevco model 2270 with grounding nut shorting its shield to the case of a single ended test unit, that becomes the common point. Test and reference charge converters should then stay isolated)  
For adhesive mounting, measure insulation and connect grounding accordingly as above | Don’t add extra ground paths ("just to be sure its grounded")  
Don’t use damaged insulating mounting studs (verify isolation)  
Don’t inadvertently ground a differential bridge by connecting to single ended conditioning (unless bridge excitation is floating) |
|   |   |   |
| **8. Signal conditioning** | Check that frequency response, slew rates, overload recovery, and filters are appropriate (check anti-aliasing requirements for digital sampling)  
Measure bridge excitation voltage at the connector (use sense lines to correct for lead resistance) | Don’t use direct voltage reading of piezoelectric devices; instead use charge converters  
Don’t use large input capacitances (such as from long cables) without knowing the effect on noise and frequency response (use remote charge converter with piezoelectric transducers in very noisy conditions) |
|   |   |   |
| **9. Excitation sources** | Be sure motion is pure, particularly at the reference frequency  
For shock or vibration, maintain alignment so motion has minimal cross axis input and rotation | For shock, don’t use pulse durations too long or too short, which have frequencies outside the useful bandwidth of either channel. If this is not possible, keep frequency response of both channels as similar as possible |
|   |   |   |
| **10. General calibration goals** | Avoid frequencies at which there is distortion due to resonances, or those at which there is more than 10% transverse motion  
Use a standard for a frequent (daily) check of system accuracy, and record trend  
Choose gain range and acceleration amplitude to provide optimum signal-to-noise ratio (use full dynamic range of data acquisition if possible)  
Control and/or record pertinent parameters, and know the effect each has on the calibration (Examples: Temperature, humidity, line voltage, grounding configuration, gain settings, etc.)  
Maintain traceability of all calibration equipment used, and establish and maintain calibration intervals | Don’t use a reference transducer above approximately 20% of its resonance without good knowledge of the effect that the mass of the test transducer and fixtures has on the frequency response (and sometimes sensitivity) of the reference transducer  
Don’t interchange equipment without knowing the effect of the change on the entire system  
Don’t hesitate to call the transducer manufacturer with performance questions, particularly to learn what parameters may affect the measure- merits, or whether there are any peculiarities (such as “minor” resonances)  
Don’t trust any measurement unless you can prove its validity and know its uncertainty |
BACKGROUND: ACCELERATION

Definitions

The position of an object is established with respect to some fixed reference, and the displacement or translation (given in meters, m) measures its change in position. If the position changes during a period of time, the object has velocity (given in meters per second, m/s). Velocity is a vector quantity, which means it is a parameter with magnitude and direction. If either the magnitude or direction of the velocity changes over a period of time, the object has experienced acceleration (meters per second squared, m/s²). The larger the velocity change, or the smaller the time over which the change occurs, the larger is the instantaneous acceleration magnitude. Acceleration is a vector quantity like velocity and displacement, also having magnitude and direction.

Momentum and force are also vector quantities. Momentum is the product of mass and velocity. Force is necessary to change the momentum of an object, either to change its magnitude or direction. The mass, M, of an object determines what acceleration a results from the net force F, by Newton’s law, F=Ma. Net force is the summation of all applied forces.

\[ F = Ma \]

An equivalent net force on a smaller mass results in a larger acceleration. If all forces balance each other, the net force is zero; there is no acceleration, and velocity remains constant.

Acceleration events can be classified in two categories, either transient or persistent. A shock or bump is a transient event, usually short-lived and having large peak acceleration values. Persistent accelerations usually are smaller, and by this definition, occur over longer durations. Vibration is described here as a persistent condition of time-varying accelerations, usually causing no net displacement when averaged over time, and involving frequent changes of direction. Compare this with free fall, a rare form of persistent, that is, nearly time-invariant acceleration which results in large displacements. Time-invariant, or steady-state, acceleration does not change magnitude or direction (it is often, erroneously, referred to as “dc” or “static” acceleration). It is of one polarity, that is, it is either positive or negative in value.

Centripetal acceleration \( a_c \) is an example of acceleration which can vary in direction but not magnitude. It is the acceleration needed to maintain circular motion, causing the paths of rotating pieces to veer in an arc, keeping them from flying off along their respective tangential paths defined by their momentum at any given instant. The direction of centripetal acceleration is rotating with the motion, always toward the center of rotation. The magnitude is constant if the rotational (or angular) velocity, \( \omega \), and the distance, R, from the center is constant, and is given by \( a_c = \omega^2 R \), where \( \omega \) is given in radians per second, rad/s. [A radian is a handy angle whose arc is the same length as its radius. There are \( \pi \) radians in a circle, and approximately 57.296 degrees in a radian.]

\[ \omega, R, \theta \]

Tangential acceleration \( a_t \) is perpendicular to that of
centripetal acceleration, also a linear acceleration (that is, pertaining to movement along a line). It is part of the definition of angular acceleration \( \alpha \) (in rad/s\(^2\)), which is the rate of change of angular velocity \( \omega \). The relation between tangential and angular acceleration is \( \alpha R \).

In the same manner that accelerating a mass requires a force, angular acceleration results from a net torque \( T \) on a given moment of inertia, \( J \), such that \( T = \alpha J \). Torque is the product of force and a moment arm, so is given in units of force times distance, and \( J \) has the dimensions of mass times distance squared.

Vibration

Vibration, whether linear or angular, may have random and/or periodic components, (in fact the total motion may include steady-state velocity and acceleration as well). In any periodic acceleration the motion repeats itself after a fixed period of time, with a frequency equal to \( 1/\text{period} \), given in Hz (cycles per second). The most basic periodic waveform is the sinusoid. Truly periodic signals are referred to as deterministic. That is, if the frequency, amplitude and phase of the components of the periodic waveform are known at any one instant, it is possible to determine the amplitude for all time, past and future.

By contrast, the amplitude of a random signal can only be described in statistical terms. The amplitude at any time is described by the probability of being within certain amplitude windows. The frequency and phase content, as well, are randomly distributed.

The acceleration amplitude of a periodic waveform is directly related to the displacement amplitude and the square of the frequency. This squared relationship has important implications in accelerometer calibration. To achieve a given acceleration at low frequency, very large displacements are needed; inversely, at high frequencies, displacements for an equivalent acceleration are tiny. For example, a sinusoidal acceleration at 1 Hz (1 cycle per second) with peak magnitude equal to the effect of a standard gravity, [one “g” = 9.80665 m/s\(^2\)], would require a peak-to-peak stroke of nearly 0.5 meter. For one g acceleration at 10 000 Hz, displacement is \( 10^9 \) smaller, only 1% of a wavelength of visible light!

Shock

A bump or shock (not to be confused with supersonic shock in compressible gases) is the time history of a transient change of momentum. The impulse of the shock is defined to be the integral over time of the net force (essentially the area under the curve of force vs. time). The impulse is equal in magnitude to the momentum change of the event.

This means that the acceleration produced by a given change in velocity can vary considerably, depending on the duration of the event. For a dropped object striking the floor, the result could be a small force spread over a long time as it squishes into a cushion, or a very large force over a very brief period as it strikes bare concrete. Considering the pulse duration to be the inverse of frequency, the relationship between amplitude and pulse duration is analogous to that between displacement and frequency in the vibration discussion above.

Descriptions of Motion

Usually, the motion of a point is described with displacements, velocities, etc. in each of three orthogonal linear directions, each perpendicular to each other, such as up-down, left-right, and forward-back. If these directions are not rotating or accelerating, they constitute an inertial reference frame.

The directions of possible motion are called degrees of freedom, or DOF’s. In any one of these directions, displacement and its derivatives, which are the rates of change, such as velocity, acceleration, jerk, etc., all are defined by the same DOF. One point bouncing around in three-dimensional space would have 3 DOF’s. If constrained to motion in a plane, the point would have 2 DOF’s. The description of the simplest system, that of motion only in a line, is a single-DOF system. Motion of a system with two independent points would be fully characterized with 6 DOF’s; three such points would
need nine DOF’s, etc.

Fully describing the motion of a body made of an infinite number of points, each with its own three DOF’s, is impossibly complex if viewed in this way. Think of the difficulty of describing the motions in a swarm of bees, a bowl of gelatin, or a tumbling rock. However, in the last case, description of the motion can be greatly simplified by introducing the concept of rotation and the idea of a rotating reference frame planted in the body and which moves with the body.

Rotation is constant throughout a rigid body. Every point experiences the same value of rotational displacement (radians, or rad) and its derivatives: Velocity (rad/s), acceleration (rad/s²), etc. Rotations are vectors, and in three-dimensional space their directions can be thought of in terms of the three linear axes that could act as axles around which the rotations occur. Figure 3 depicts the orientations of positive rotations around three orthogonal linear axes using the right-hand convention.

![Figure 3](orientation_of_axes.png) Orientation of axes. Arrows show directions of translations and rotations which are positive in the right-hand convention.

If the rotating body is rigid, defining just the three rotational DOF’s helps to describe motion of all points. To complete the description of motion, we then assign three more linear motion DOF’s. Analytically, the most convenient location is the center of gravity (e.g.) about which the rotations occur, although the choice of a reference location is arbitrary. Practical reference points are more likely to be at the surface, not within the body. In any case, knowing the six DOF’s and the position of a point relative to the e.g., the motion of any point can be known in a very efficient manner.

These six DOF’s are sufficient to describe the motion of the entire object only if the object is rigid, that is, if there is no bending or other strains causing relative motion within the object. Of course nothing is truly rigid. An actual continuous body has an infinite number of ways it can vibrate. But modeling real systems this way is seldom successful, since the analytical solutions to the equations of motion are known only for a few classes of simple shapes.

Two ways to simplify models with relative motion between parts are lumped parameter models, and finite element analyses (FEA’s). A large elastic model can be lumped into supposedly rigid pieces, each with its own c.g., interconnected by springs and dampers. These lumped models differ from FEA’s, which break the object into many connected elastic pieces of simple shapes, for each of which the analytical solutions are known.

To fully study the motion of a given part of the structure, at least as many accelerometers (or other motion-measurement devices) would be needed as there are DOF’s. Channels devoted to redundant measurements and off-axis directions are advised as a check of model consistency. Despite how the model of an object is simplified, the number of channels usually is large. The practical user must determine which of the possible motions are worth monitoring, a topic outside the scope of this paper.

Also outside the scope is the measurement of angular acceleration and the calibration of angular transducers. Although uncommon in shock and vibration testing, angular accelerometers do exist [6]. Usually, however, an angular acceleration measurement is done by taking the difference of the outputs of linear accelerometers, taking into account their orientation and spacing [7].
BACKGROUND: ACCELEROMETERS

An accelerometer is designed to track the motion at the mounting surface with fidelity. Ideally, not only should its output correspond to the surface motion exactly, but the presence of the transducer should not modify the motion to be measured. As will be described, both goals are impossible, but are approximately true between certain frequencies. The degree to which the approximation is true depends on the masses and stiffnesses of both the accelerometer and the structure to which it is mounted.

Single-DOF Model

The simplest description of an accelerometer is a single-DOF system (per above, an object constrained to motion in only one direction) which produces an output proportional to the component of acceleration in its sensitive direction. It is idealized in Figure 4 as a mass connected to a mounting surface only by a one-dimensional spring and a dissipative element called a damper. The system is described as a second order system. The differential equation representing the system relates the displacement of the mass (with respect to the mounting surface) to its first and second derivatives with respect to time, which are velocity and acceleration, respectively.

Accelerating the mass requires a force. Ignoring for now the damper and body forces such as gravity, the force $F$ comes from the spring, compressed by an amount $x$ between the inertia of the mass and the accelerating mounting surface. The deflection is then proportional to the acceleration of the mass, to the extent that the stiffness $k$ of the spring remains a constant in Hooke’s law $F=k x$.

The “spring” usually includes materials which either produce or modulate an electrical signal, such as piezoelectric ceramics depicted in Figure 5, or piezoresistive strain gages in Figure 6. (In another design the geometry of the system is such that the deflection changes a capacitance.)

The “damper” of the single DOF system is usually embodied in the intrinsic internal friction of the materials and joints, although occasionally it is augmented by viscosity in the flow of fluid due to motion of the components.

The “mass” is commonly a piece of dense metal bolted to the base through the spring. The higher the value of mass, the higher is the sensitivity and lower are the natural frequency (described below), full scale range and over-range limit. Full scale is the acceleration at which the linearity of the transducer stays within acceptable values, and the over-range limit is the level at which generated stresses could cause damage.

All the mechanical elements have mass, including the
spring elements, contributing to the overall proof mass. The system behaves as if that mass were concentrated at a point, called the inertial center of mass, the center of seismic mass, or the inertial center of gravity. For analysis of motions which include rotations and/or angular acceleration, this is the reference point for the transducer response.

![Figure 6 Typical piezoresistive accelerometer. The cantilever design is an example of a single DOF system in which the degree of freedom can be described as an angle. The example shows how interchangeable linear and angular motions and descriptions can be. Because the center of mass is offset by the moment arm L from the direction of applied acceleration, a torque is required at the base of the beam to generate the force to accelerate the mass, (in addition to the torque to provide the angular acceleration to the mass' moment of inertia). This torque comes from two sources: The bending of the beam by the angle \( \theta \), and from the stretching of the gages by an amount \( \theta \text{t} \) (the forces from which are multiplied by the moment arm \( \text{t} \)). For the polarity of acceleration shown, the gages above the neutral axis are in tension, and the gages below the neutral axis are in compression.

The location of the seismic center may not be obvious from the design. In PE shock accelerometers, for example, the inherent mass in the sensing element itself is often sufficient for required sensitivity, so no part called “mass” is added. In some piezoresistive designs the entire spring-mass structure is etched in miniature from a silicon wafer. Whatever the geometry, virtually all accelerometers can be modeled as a system with these spring-mass-damper elements.

If you give a mechanical pulse to any such system with a relatively low value of damping and then let it respond freely, it will oscillate with a characteristic frequency, called the resonance or natural frequency \( f_n \). The oscillations will dissipate slowly with low damping, more quickly with larger values. The ratio between the stiffness of the spring and the value of mass determines \( f_n \). For the single DOF system the value of the resonance frequency is

\[
 f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}}
\]

![Figure 7 Decay of free oscillation. The amplitude of a freely oscillating single-DOF system decays exponentially with a time constant which depends on the damping. The damping value of the lower curve is five times larger than that of the upper curve.]

**Amplitude Response**

The solution of the response of an idealized single-DOF model is the notoriously frequency dependent relation shown in Equation 1 and Figure 8. The ratio of output to input amplitudes as a function of frequency \( f \) is given by

\[
 \frac{\text{Output}}{\text{Input}} \approx \frac{S}{\sqrt{\left(1 - \left(\frac{f}{f_n}\right)^2\right)^2 + \left(2\zeta f/f_n\right)^2}}
\]

where \( S \) is the sensitivity, and the damping ratio is \( \zeta \).

![Figure 8 Amplitude response of a single-DOF system vs. frequency. The amplitudes have been normalized (divided) by the sensitivity. Frequencies have been normalized by the natural frequency. Values of the damping ratios in the six curves are given as a fraction of critical damping. Critical damping is the value below which the response of the system is oscillatory. At damping equal to \( \sqrt{2} \) (\( \zeta \) of approximately 0.7) response is optimally flat.]
Driving a system at or near its $f_n$, can result in very large amplitudes, as shown by the large peaks for the underdamped conditions (those for which the value of $\zeta$ is much less than the critical value of 1). Drive the system above $f_n$, and the amplitudes are strongly attenuated. In neither of these frequency bands are accelerometers very useful. Mercifully, at frequencies low compared to $f_n$, the relationship between input amplitude and response amplitude is not strongly a function of frequency.

This low frequency range is the operating range of an accelerometer, (known as its passband, up to between $.2$ and $.5 f_n$, depending on damping ratio $\zeta$.) As frequency ratio approaches zero, (ignoring for now the low frequency effects of ac-coupled response), the deviation from $S$ becomes vanishingly small. The width of the passband is a trade-off with the sensitivity, as illustrated in Figure 9.

Figure 9  Two single DOF systems with equivalent spring constants and damping ratios, but one with mass 10 times that of the other, would have the comparative responses shown. Since accelerometer sensitivity is proportional to the value of its mass, the upper curve represents ten times the sensitivity, and its resonance is lower by the square root of ten, just over 3 times smaller.

**Boundary Conditions and Multiple DOF’s**

As in simple single-DOF systems, the resonances of multiple-DOF systems are also determined by the stiffnesses and masses of the components. With more components and/or possible directions of motion, the equations predicting the resonance values for multiple DOF systems are more complex, but generally the largest masses and weakest springs dominate the expressions. The relationships are similar to the single DOF systems; for example adding mass lowers resonance frequencies.

Adding an accelerometer to a system will therefore modify the motion to be measured. The transducer adds mass to the structure to be measured (and sometimes it adds stiffness, as well), changing the structure’s resonances and hence its response. Conversely, the resonance of the accelerometer can be affected by the structure to which it is mounted.

As an example of this latter effect, consider the extremes of the possible mounting conditions of the transducer. That of the mounted and unmounted state. In the simplest case, the mounting surface is considered rigid and massive, a fixed boundary condition at one end of the spring. At that boundary is a node, a point with essentially no motion during free vibration.

The opposite case is a free boundary condition. That is, if the accelerometer is unmounted, (or is mounted to a very light structure) the base of the transducer becomes a dynamic component rather than being fixed or an insignificant part of the mounting structure. For accelerometers with bases similar in size to the active “mass”, this can result in significantly different mounted and unmounted resonances. This is illustrated in Figure 10.
Changes in boundary conditions can change the resonance frequency of an accelerometer. In this example the mass of the transducer “mass” is equal to that of the base. In the case of free vibration in the unmounted condition, the “mass” and the base would move with equal velocity but opposite direction about the node. The spring is effectively shorter and stiffer, and the resonance is higher, relative to the fixed boundary condition.

One explanation comes by observing that every action has an equal and opposite reaction. As shown in Figure 10, momentum of the base is counteracted by that of the mass, moving in the opposite direction during free vibration in the unmounted condition. At the node between them, the system behaves as if that point were fixed. The node is in the spring, so the spring is effectively shorter and stiffer, and the resonance is higher, than when the transducer is mounted.

Motion of the base actually occurs in the “fixed” condition as well; the massive mounting surface also moves opposite to the transducer mass in free vibration. Its velocity is simply proportionately smaller due to its greater mass, and the node is nearer the mounting surface. The “fixed” or “free” nature of the boundary condition is all a matter of degree.

In calibration, the stiffness and mass of the shaker armature or the shock anvil are designed to be a practical approximation to the fixed condition for the frequencies of interest. Generally, transducer resonances will change only slightly with differing mounting conditions, such as due to the stiffness and density differences in aluminum vs. steel fixtures.

However slight, the effect exists, and illustrates that mounting surfaces should be thought of as a component of the system, and that the base of the transducer (and perhaps a mounting fixture) are all spring/mass components of a complex system, as illustrated in Figure 11. These are examples of multiple-DOF systems. Complications come from the fact that there will be as many natural frequencies as there are degrees of freedom.

The mounting structure is not strictly a fixed boundary condition. The structure, the transducer, and any mounting fixtures should all be considered dynamic parts of the overall system, each with their own degrees of freedom, and therefore, each with their own natural frequencies. As depicted, the surface and the fixture are more massive and stiffer than the components of the transducer, but this is not necessarily the case.

Associated with each natural frequency is a mode shape, the pattern of displacements each DOF takes on in that mode. The simplest shape is the lowest frequency (or fundamental) mode, in which all masses move with the same polarity of motion. Higher modes occur at higher frequencies and have more complicated combinations of motion, such as a mounting fixture moving opposite to the transducer.

Comparison calibrations can only be made at frequencies in which the fundamental mode is dominant (or said another way, at frequencies at which higher modes contribute insignificantly to overall motion). As will be described later, the comparison is made to a reference transducer usually buried in the mounting surface. Both transducers and all other components must be moving in concert for the comparison to have relevance. This will occur at frequencies below the fundamental resonance. If test frequencies are far enough below the higher modes, a single-DOF description using the fundamental resonance is often adequate, despite the actual complexity of the system.

**Phase Response**

Returning to the topic of single-DOF response, not
only are amplitudes modified by accelerometers and other structures, there is a time delay between the acceleration to be measured and the resultant output. The time delay is a function of the frequency content of the input and the damping of the transducer (remember this discussion is of the transducer only; it must be noted that signal conditioning adds its own phase and amplitude characteristics, particularly if filtering is included). For sinusoidal input of a particular frequency, the time delay is a phase angle $\Phi$, given by Equation 2 and shown in Figure 12.

$$\tan \Phi = \frac{2\zeta \frac{f}{f_n}}{1-(\frac{f}{f_n})^2}$$

where $\Phi/2\pi f$ is the number of seconds that the measurements lags behind the actual acceleration (if $f$ is in Hz and $\Phi$ is in radians).

For $0^\circ$ phase, there is no delay, which is approximately the case of underdamped accelerometers. For optimally flat damping, $\zeta = 0.7$, it turns out that the phase is approximately linear, that is, the phase angle is proportional to frequency, which results in a constant time delay for any frequency. This is particularly important in the case of a complex input such as shock, which is made up of many frequencies. The shape of the waveform would not be maintained if the higher frequency components, for instance, were subject to more delay in the measurement than lower frequency components.

**Resonant Excitation**

If a lightly damped system is driven near its resonance, that is, if it is given acceleration at the natural frequency, the amplitude builds up with each cycle. The deflecting spring and moving mass alternately store the input energy as potential and kinetic energies, respectively.

At precisely the natural frequency, amplitude initially increases at a rate of $\pi$ times the input amplitude per period [8]. For example, after 10 periods, a one g input would be amplified to 31.4 g. (This would take only 1 millisecond for $f_n = 10$ kHz!) Eventually the amplitude would asymptotically approach a steady-state level at which the energy dissipated per cycle is equal to the energy input. The time constant of the amplitude “envelope” in a resonant build-up is precisely the same as in the subsequent free decay of the oscillation when excitation ceases.

Driving at resonance is the same as inserting $f=f_n$ in equation (1) above, resulting in an output equal to $(1/2)\zeta$ times the input. The value of $\zeta$ for many accelerometers can be as low as .01, so with an amplification factor of 50, just a few g’s input can build up to hundreds of g’s of actual acceleration in the transducer. Care is advised when shaking an accelerometer at frequencies at or near its natural frequency.

**Stress Waves**

It takes time for acceleration at the mounting surface to travel through the spring to the mass. (This can be related to the phase described above, but has other ramifications, as described.) The force applied to the area of the base creates a pressure disturbance, propagating through the material. For many structural materials the velocity of the disturbance, its “speed of sound”, is about 5000 m/s for the compression wave. (Just as in the study of earth quakes, there is a shear wave which is slower.) Therefore it takes a microsecond
for the compressive stress wave to traverse 5 mm of transducer. This is followed by the shear waves and reflections off discontinuities in the geometry, converting the original disturbance into other directions of motion.

Such considerations are important for understanding the timing of some shock conditions, but more directly help the realistic visualization of “rigid” structures as the gelatinous blobs they are, capable of wiggling in any direction.

**Minor Resonances**

From the statements above, it should not be a surprise that accelerometers are not perfect single degree of freedom systems. They are generally complex structures, with peripheral pieces that have several DOE’s of their own. Covers, leadwires, connector assemblies, etc. can all resonate at independent frequencies, adding their sometimes-not-so-feeble forces to the sensor. If these vibrations occur in the sensitive direction, they can cause “glitches” in both the frequency response curves and the life of the accelerometer designer. These are sometimes referred to as “minor” resonances.

Manufacturing tolerances and sensor element variations tilt the sensitive axis by an angle θ, placing a component with magnitude equal to sinθ in the transverse plane. Alignment is critical. Only 1° misalignment creates a transverse component 1.74% of the magnitude of the sensitive axis.

**Transverse Sensitivity**

An accelerometer should respond only to accelerations along an ideal input reference axis defined by its geometry, (usually the direction perpendicular to the mounting surface), and should have zero sensitivity in any directions transverse (orthogonal) to that ideal direction. A common imperfection in accelerometers is misalignment of the sensitive or input axis [the direction of acceleration causing maximum output] from the ideal [see Figure 14]. This places a component of the sensitivity vector in a direction transverse to the ideal axis, and therefore, gives the transducer transverse or cross-axis sensitivity.

Transverse outputs are maximum when the transverse input is aligned with the direction of the tilt. [Note that accelerations in the mounting plane along the line 90° from the tilt would result in no output.] Transverse sensitivity is the ratio of the maximum transverse output to the magnitude of transverse input, divided by the sensitivity in the ideal direction.

**TRANSUCER TYPES:**

The most common accelerometer types are 1) piezoelectric (PE), 2) PE accelerometers with integral electronics (IEPE), 3) piezoresistive (PR), 4) variable capacitance (VC) and 5) servo feedback. Each has its own strengths and weaknesses, and serves in differing applications. This section of the paper describes each type and addresses issues that affect the measurement...
uncertainty of calibrating these accelerometers.

**Piezoelectric Accelerometers**

The output of the “self-generating” sensors in PE accelerometers is charge. That is, the “spring” sensing elements provide a given number of electrons proportional to the amount of applied stress (piezein is a Greek word meaning to press). There are many natural and man-made materials which display this characteristic, usually in the form of crystals or ceramic, although some are polymers. All such materials have a regular crystalline molecular structure, with a net charge distribution which changes when strained.

Figure 15  Piezoelectric materials are usually made of ions oriented in a crystal cell, which has a structure in which a net change in the separation of the positive and negative charges occurs when the cell is strained. The direction of the separation (which, depending on the crystal, is not necessarily in the direction of the applied force) defines the surfaces on which the electrical signal appears. Compression sensitivity is shown on the left, (an example of a linear DOF). This example would not develop net charge on the sides for the given strain. On the right depicts the result of shearing stresses (a torsional DOF).

One property all piezoelectric materials share is that they have no center of symmetry in the unit cell of the structure [9]. Deforming a symmetric cell produces no electric fields.

Piezoelectric materials may also have a dipole (which is the net separation of positive and negative charge along a particular crystal direction) when unstressed. In such materials, fields can be generated by deformation from stress or temperature, causing piezoelectric or pyroelectric output, respectively. The effects are reciprocal. The PF sensor can be made to distort with the application of a field or temperature. PE sensors without a dipole (such as quartz) are not pyroelectric.

The pyroelectric outputs can be very large unwanted signals, generally occurring over the long time periods associated with most temperature changes. They might not be noticed if the signal conditioner does not respond to those low frequencies, but they can be very large as thermal rates and gradients increase. (The polymer PE materials have such high pyroelectric output that they are often used as thermal detectors.) Some types of PE materials also produce very high frequency “glitches” in response to thermal transients, that can be easily mistaken for acceleration events.

Charges are actually not “generated”, just displaced. (Like energy and momentum, charge is always conserved.) What is generated is an electric field along the direction of the dipole. Metallic electrodes on faces at the opposite extremes of the gradient provide mobile electrons, which move from one face, through the signal conditioning, to the other side of the sensor to cancel the generated field. The quantity of electrons depends on the voltage created and the capacitance between the electrodes. A common unit of charge from an accelerometer is the picocoulomb, or $10^{-12}$ Coulomb, which is slightly over 6 million electrons.

The position of the electrodes depends on the crystal orientation and the type of stress to be sensed. A given material may be sensitive to several directions and types of strain. (Note that the two simplified examples of Figure 15, showing compressions and shear strains, corresponding to linear and rotational DOF’s, are the same crystal material, simply rotated 45°.) It is important to note that the alignment of the sensitive axis of the accelerometer (affecting cross-axis sensitivity) is only as good as the net alignment of the dipoles with respect to the faces.

Many types of PE materials are used, with the choice being a trade-off of charge sensitivity, dielectric coefficient (which, with geometry, determines the capacitance), thermal coefficients, maximum temperature, frequency characteristics and stability. The best signal-to-noise ratios generally will come from the highest piezoelectric coefficients.
Naturally occurring piezoelectric crystals, such as tourmaline or quartz, generally have low charge sensitivity, nearly one hundred times smaller than that of the more commonly used ferroelectric materials. Allowing smaller size for a given sensitivity, ferroelectric materials are usually man-made ceramics in which the crystalline domains (regions in which dipoles are naturally aligned) are themselves aligned by a process of artificial polarization.

Polarization usually occurs at temperatures very much higher than operating temperatures, to speed the process of alignment of the domains. Depolarization, or relaxation, can occur at lower temperatures, but at very much lower rates and can also occur with applied voltages and preload pressures. Depolarization always results in loss of sensitivity. Tourmaline is a natural crystal which does not suffer such depolarization, used particularly at very high temperatures.

PE accelerometers are called self-generating transducers. They produce an electrical output signal without the use of an external electrical power source. Nothing comes free in nature, of course; they transduce mechanical strain energy in the ceramic into electrical energy in the circuit, hence the generic name “transducer”, actually drawing a small amount of energy from the mounting surface. An important consequence of this is that PE transducers cannot be used to measure steady-state accelerations.

Such an acceleration would put a fixed amount of energy into the crystal and a fixed number of electrons at the electrodes. Conventional voltage measurement would bleed electrons away, as does the internal resistance of the sensor. (High temperature or humidity in the transducer will make things worse by reducing the resistance value). Energy would be drained and the output would decay despite the constant input acceleration.

Measuring the voltage of a PE transducer is generally a bad idea, unless the signal conditioning is within the transducer, see next section. The piezoelectric transducer is effectively a voltage source driving a series capacitor C, producing a charge Q across its plates according to \( V = Q / C \). Making a measurement requires connecting the sensor to a meter through cables. Cable and meter input capacitance is in parallel with the transducer. The voltage \( V_i \) decreases as total input capacitance increases, proportional to the ratio of the sensor capacitance to the total capacitance.

Voltage measurements of PE transducers are impractical because the measurement amplitude is then directly dependent on the cable and system capacitances. Input impedance of the amplifier would also directly affect the response at low frequencies, possibly raising the high-pass corner into the frequency band of interest. (The list goes on and on. Use of voltage amplifiers to measure PE output is not recommended.)
Figure 17 Equivalent circuit of a PE sensor and a charge convertor. The charge output of a sensor is equivalent to a voltage source driving a series capacitor. The parallel internal resistance of the sensor drains off the electrons, preventing dc response. Because the net output of the sensor is charge, the voltage output is shunted by the parallel capacitance of a cable. A charge convertor is not affected by such input capacitance. It is a high impedance operational amplifier which drives a feedback capacitor with a charge equal to \(-Q\) (balancing the input \(Q\), since effectively no current flows into the input node), resulting in output voltage \(V_o = \frac{Q}{C_f}\).

Instead of measuring voltage, a charge should be measured with a charge convertor, as shown in Figure 17. It is a high impedance operational amplifier with a capacitor as its feedback. Its output is proportional to the charge at the input and the feedback capacitor only, and is nearly unaffected by the input capacitance of the transducer or attached cables. The high-pass corner frequency is set by the feedback capacitor and resistor in a charge convertor, not the transducer characteristics. (The transducer resistance changes noise characteristics, not the frequency [\& Section VI].) As with the PE transducers, charge convertor outputs are ac or capacitively coupled, (where “ac” is alternating current; that is, they do not pass steady-state dc direct current values). At low frequencies there is significant attenuation and phase shifts. Depending on application, this is not necessarily a limitation. Practically speaking, no acceleration lasts indefinitely. If time constants are long enough, the ac-coupled transducer will suffice.

PE transducers are generally extremely rugged. Designs have been refined over many decades to optimize sensitivity and bandwidth and to reduce thermal and strain sensitivities. However, there are several aspects that require care in use or interpretation of results.

1) Apparent low frequency response can be affected by zero shifting in extreme shock [10].
2) Some PE materials display a “droop” in frequency response of charge output.

Figure 18 “P8 Droop”. Some ferroelectric sensors have a frequency-dependent dielectric coefficient, resulting in a sag in the frequency response of a percent or two per decade of frequency.

3) Thermal transients can saturate the high-pass input filters of charge convertors. Some pyroelectric outputs are large enough to depolarize an unconnected ferroelectric PE transducer after a large temperature excursion. (Moral: Keep such transducers connected to signal conditioning during temperature changes.)
4) Perhaps the most important limitation of PE transducers is the requirement that they be used with “noise treated” cables. Otherwise, motion in the cable can displace triboelectric charge which adds to the charge measured by the charge convertor (described later).

Integral Electronics PE Accelerometers
Many piezoelectric accelerometers include integral miniature hybrid amplifiers, which, among other advantages, eliminate the requirement of noise treated cable. Most require an external constant current power source. Both the input supply current and output signal are carried over the same two-wire cable.

IEPE design offers several benefits. Its low impedance output provides relative immunity to the effects of poor cable insulation resistance, triboelectric noise, and stray...
signal pickup. Output-to-weight ratio of IEPE is higher than with PE transducers. Additional functions can be incorporated into the electronics, including filters, overload protection, self identification, etc.

Lower cost cable and conditioning can be used, since the conditioning requirements are comparatively lax, compared to PE or PR, (described below). Unlike PR, sensitivity of IEPE accelerometers is not significantly affected by supply changes. Dynamic range, instead, (the total possible swing of the output voltage) is affected by bias and compliance voltages. Only with large variations in current supply would there be problems with frequency response when driving high capacitance loads.

A disadvantage is that the electronics generally limit the transducer to narrower temperature extremes. Also, although generally not a problem for calibration, the necessarily small size of the amplifier may preclude the more sophisticated features of a full-blown laboratory amplifier. Slew limiting, or sometimes high output impedance, is also a concern with these transducers when driving long cables or other capacitive loads. It can often be remedied by increasing the amount of drive current.

The circuits do not necessarily need to be charge convertors, since the capacitance due to leads between the sensor and the amplifier is small and well controlled. Quartz is used in the voltage node, that is, with source followers, because its small dielectric coefficient provides comparatively high voltage per unit charge. Voltage conversion also aids ferroelectric ceramics which have the sag in frequency response in charge mode described above. The amplitude frequency response in the voltage mode is quite flat.

Piezoresistive Accelerometers

A PR accelerometer is a Wheatstone bridge of resistors, incorporating one or more legs which change value when strained. Because the sensors are supplied with energy externally, the output can be meaningfully-dc coupled to respond to steady-state conditions. Data on steady-state accelerations comes at a cost, however. Sensitivity of a bridge varies almost directly with the input excitation voltage, requiring a highly stable and quiet excitation supply.

Stability requirements for the PR transducer and its conditioning are considerably tighter than for IEPE. Low impedance PR transducers share the advantages of noise immunity provided by IEPE. (4, Section VI), although the output impedance of PR is often large enough that it cannot drive large capacitive loads. Like an underdriven IEPE, this results in a low-pass filter on the output, limiting high frequency response.

The sensitivity of a strain gage comes from both the elastic response of its structure and the resistivity of the material. Wire, thick- or thin-film resistors are of the class which have low gage factors, that is, the ratio of resistance change to the strain is small. Their response is dominated by the elastic response. They are effectively homogeneous blocks of material with resistivity of nearly constant value. As with any resistor, they have a value proportional to length and inversely proportional to cross-sectional area. If a conventional material is

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![Wheatstone bridge](image-url)
stretched, its area reduces while the length increases. Both effects increase resistance. The Poisson ratio defines the amount a lateral dimension is thinned compared to the amount the longitudinal dimension is stretched. Given a Poisson ratio of .3 (a common value) the gage factor would be 1.6; resistance would change 1.6 times more than it is strained.

The response of strain gages with higher gage factors is dominated by the piezoresistive effect, which is the change of resistivity with strain. Semiconductor materials exhibit this effect, which, like piezoelectricity, is strongly a function of crystal orientation. Like other semiconductor properties, it is also a strong function of dopant concentration and temperature. Gage factors near 100 are common for silicon gages, and when combined with small size and the stress-concentrating geometries of anisotropically etched silicon, the efficiency of the silicon PR transducer is very impressive. (Consider the stiffness-to-mass ratio of an ant. The miniaturization allows natural frequencies in some PR shock accelerometers in excess of 1 MHz.) However, often accompanying the high piezoresistive coefficients are high thermal coefficients of resistance and sensitivity.

Sensitivity deviations can be corrected by incorporating resistors of differing temperature coefficients in parallel or series, as required, to control the effective bridge temperature coefficient. Resistors in series with the input create a divider to control the amount of excitation voltage that gets to the bridge. In parallel with the output, compensation resistors shunt the output. Each technique reduces the amplitude of the output.

Compensation can also be used for the balance of the bridge, again being placed in parallel or series with any of the legs, correcting for the matching of either the resistance values and/or the change of the values with temperature.

Compensation is an art, and can have non-linear characteristics, so it is not advised to operate a PR transducer with excitation different than the conditions under which it was manufactured or calibrated. As an example, PR sensitivity is only approximately proportional to excitation, which is usually a constant voltage (or, in some cases, constant current, which has some performance advantages).

The repeatability of a calibration is a function of the repeatability in the excitation level used. Because thermal performance will in general change with excitation voltage, there is not a precise proportionality between sensitivity and excitation. Another precaution in dealing with voltage-driven bridges, those with low resistance particularly, is to verify that the bridge gets the proper excitation. The series resistance of the input leadwires acts as a voltage divider. Take care that the input lead wires have low resistance, or that a six-wire measurement be made [with sense lines at the bridge to allow the excitation to be adjusted] so the bridge gets the proper excitation.

Figure 20 Sense lines. To correct for the voltage drop across the excitation leads, some bridge conditioners use sense lines to adjust excitation until the reading at the bridge is as desired. Caution: If these leads are left disconnected, the sensing circuit will apply maximum voltage. (Output lines would have the same series resistance, but because the input impedance of the amplifier is much larger than the bridge, current through the leads, and therefore any voltage drop, is negligible.)

Constant current excitation does not have this problem with series resistance. However, PR transducers are generally compensated assuming constant voltage excitation, and may not give the desired performance with constant current.

Underdamped PR accelerometers tend to be less rugged than PE. Single-crystal silicon can have extraordinary
yield strength, particularly with high strain rates, but it is a brittle material nonetheless. Internal friction in silicon is very low, so resonant amplification can be higher than for PE transducers. Both these features contribute to its comparative fragility, although if properly designed and installed they are used with regularity to shocks well above 100,000 g. They generally have wider bandwidths than PE transducers (comparing models of similar full scale range), as well as smaller nonlinearities, zero shifting and hysteresis characteristics. Because they have dc response, they are used when long duration measurements are to be made.

The balance of the PR bridge is its most sensitive measure of health, and usually is the dominant feature in the total uncertainty of the transducer. The balance, sometimes called bias or ZMO (Zero Measurand Output, the output with 0 g), can be changed by several effects: Usually thermal characteristics, or internally- or externally-induced shifts in strains in the sensors. Transducer case designs attempt to isolate the sensors from external strains, such as thermal transients, base strain or mounting torque (described below). Internal strain changes, such as epoxy creep, tend to contribute to long term instabilities. All these generally low-frequency effects are more important for dc transducers than the ac-coupled devices for obvious reasons, occurring more often in the wider frequency band of the de-coupled transducer.

Some PR designs, particularly high sensitivity transducers, are designed with damping to extend frequency range and overrange capability. Damping coefficients of approximately 0.7 are considered ideal (see Figure 8). Such designs often utilize a viscous fluid such as oil. Two characteristics dictate that the technique is useful only at relatively low frequencies: that damping forces are proportional to flow velocity, and that adequate flow velocity is attained by pumping the fluid with large displacements. This is a happy coincidence for sensitive transducers, since they operate at the low acceleration frequencies at which displacements are adequately large.

Viscous damping can effectively eliminate resonant amplification, extend the overrange capability, and can more than double the useful bandwidth. However, because the viscosity of the damping fluid is a strong function of temperature, the useful temperature range of the transducer is substantially limited. More important to the subject of calibration, the frequency response becomes extremely temperature sensitive: A warm shaker can give quite different frequency response results for an oil-filled PR than one which is cool. [Here is a good example of why the reported temperature of a calibration should be based on a measurement at the transducer, not just that of the laboratory.]

**Variable Capacitance Accelerometers**

VC acceleration sensors are usually designed as parallel plate air gap capacitors, in which motion is perpendicular to the plate. In some designs the plate is cantilevered from one edge, so motion is actually rotation, whereas others are supported around the periphery, as in a trampoline. Many are manufactured as a sandwich of anisotropically-etched silicon wafers. The micromachining allows a gap of only a few microns thick, so they can be designed with air damping. The fact that air viscosity changes by just a few percent over wide operating temperatures provides stable frequency response compared to oil-damped PR designs.

The incorporation of overtravel stops in the gap can enhance ruggedness in the sensitive direction, although resistance to overrange in transverse directions must rely solely on the strength of the suspension, [as with all other transducer designs which do not have overtravel stops]. Extremely high acceleration overrange conditions can be survived with some designs, literally 1000 times full scale range [11]. VC’s can provide many of the best features of the transducer types above: Large overrange, dc response, low impedance output, and simple external signal conditioning.

A disadvantage is the cost and size of increased complexity of the on-board conditioning. Also, high
frequency capacitance detection circuits are used, and some of the high-frequency carrier usually appears on the output signal. It is generally not even noticed, being up to three orders of magnitude (that is, three powers of 10, or 1000 times) higher in frequency than the output signals.

**Servo Accelerometers**

All of the accelerometer types described above are open-loop accelerometers, in which the output due to deflection of the sensing element is read directly. In servo-controlled, or closed-loop accelerometers, the deflection signal is used as feedback in a circuit which physically drives the mass back to the equilibrium position. Force is provided usually by driving current through coils on the mass in the presence of a magnetic field. Output is the applied drive current itself, which, analogous to the deflection in the open-loop transducers, is proportional to the applied force, and therefore to the acceleration.

In contrast to the rugged spring elements of the open-loop transducers, the restoring force in the case of the closed-loop accelerometer is primarily electrical (and exists only when power is provided). The springs are made as flimsy in the sensitive direction as possible. Likewise, most damping is provided through the electronics.

Servo designs tend to have better bias stability, cost more, and be much more fragile than open-loop transducers. They generally have a useful bandwidth of only approximately 100 Hz, and are designed for use in applications with comparatively low acceleration levels and extremely low frequency components.

Calibration of servo accelerometers is usually performed using gravity as the source of excitation. This contributes to the accuracy associated with servo accelerometers, given the astonishing degree of precision with which local gravity can be measured, as described later. (The smallest uncertainty for transducers calibrated by comparison to an ac-coupled reference transducer are at best a few parts per thousand, contrasted with uncertainties of only parts per million for values of local gravity.)

**GRAVITY, "NET-FORCE-OMETERS", AND RELATIVITY**

There is an important point to be made about how any accelerometer responds to gravity. In Newtonian physics, remember that gravity is an attractive force between masses, proportional to the masses involved, and inversely proportional to the square of the distance between them. The dominant gravitational force we experience is that of the earth (the largest nearby mass); lesser forces are due to the moon and sun, affecting tides, for example. The force of gravity would accelerate an object toward the combined center of gravitation if unopposed.

As described previously, the output of an accelerometer corresponds to the force that causes deflection in the spring, not necessarily acceleration. An accelerometer will respond to gravity even if it is not accelerating. The transducer responds equivalently to an acceleration or gravitational force that is being opposed.

Newton concluded that acceleration is the result of the net force acting on a mass. As an example, gravity is now forcing you and your chair into the floor. You are not accelerating, the floor and the earth beneath it are deflecting and pushing back with equal upward force. Net force on you is zero, and so is your acceleration. You would accelerate downward if the earth opens up and the floor breaks through. While you fell, you would feel no force, [neglecting friction and sudden decelerations, of course]. At the same time, a dc-coupled accelerometer would register a 1 g output while the floor was intact, and 0 g while falling free [which is odd, considering it is accelerating]. An ac-coupled device would sense a change of ~ 1 g at the beginning of free fall as the stress on its sensor suddenly decreased. Its output would then decay to zero during free fall, as it would in any long-duration [low frequency] condition.
A conclusion of this discussion could be that, to be semantically correct, accelerometers should be called “net-force-ometers.” They respond to net force on their mass, whether the source is acceleration, gravitation, sound pressure, magnetic fields, etc.

Einstein would explain that our notions of gravity and true inertial reference frames are mistaken, and that visual clues of relative motion fool us into believing that there is a difference between acceleration and gravity. Actually there is simply no way to distinguish between uniform acceleration and a constant gravitational field. They really are the same phenomenon, he says, explained by the warpage of space-time due to the presence of mass. [Reference 12 provides a very readable explanation.]

Relativistic physics may not be a common topic in calibration, but the discussions above should at least raise a caution flag when interpreting the output of “net-force-ometers” when gravitation is a significant contributor to input. A simple example is the orientation of a shaker in gravity when testing a 2 g full scale unit. Using 2 g sinusoidal acceleration, with the shaker oriented up, gives an actual force on the accelerometer equivalent to an acceleration range from -1 to +3 g. Linearity in this range may differ from that of the desired test range of -2 to +2 g. In such a case it might be better to orient the shaker in the horizontal position, so gravity is a transverse input.

CALIBRATION REQUIREMENTS

The military standard provides requirements for the establishment and maintenance of a calibration system, to include, among other things:

1) Written system description
2) Traceability and accuracy limits
3) Environmental controls
4) Intervals of calibration
5) Calibration procedures
6) Procedure for out-of-tolerance conditions
7) Procedure for establishing system adequacy
8) Documentation of traceability
9) Records showing procedures are followed
10) Calibration status labeling

Similar requirements are established by ISO 9001 [4.11], such that the calibration program include:

a) Identification of equipment to be calibrated
b) System to ensure calibration intervals
c) Records of traceability
d) System for checking previous test results if equipment is found to be out of calibration

METHODS OF ACCELEROMETER CALIBRATION

This paper focuses on secondary calibrations, which are based on comparison of the unit under test (UUT) to a secondary standard, or a reference transducer (the terms are used here interchangeably).

The secondary standard is usually given a primary or absolute calibration in its recalibration cycle, or is itself compared to a transfer standard which was calibrated by a primary method. Only quantities such as mass, length, time and voltage, which are defined absolutely, are used in primary calibrations.

The following section summarizes many of the calibration methods described in the International Standards Organization’s ISO 5347-0 “Methods for the Calibration of Vibration and Shock Pick-ups”. Unique features which contribute to uncertainty are described,
and sources of uncertainty which are more common among all techniques are given in the next section.

**Primary Vibration Calibrations**

**Optical Displacement Measurement**
The simplest primary vibration calibration is by the optical measurement of peak-to-peak displacement. The blur of motion is visually measured against a ruler using a measuring microscope, and knowing frequency, acceleration is calculated. Relatively large displacements are necessary for adequate resolution.

**Laser interferometry**
The Helium-Neon laser provides a stable wavelength of light which is an accepted primary definition of length, equal to approximately 0.633 micrometer. Laser interferometry uses the fringe patterns set up in space when beams from the same source interfere with each other. Usually a mirror is placed on the surface to be monitored, reflecting the light and creating a moving interference pattern. Provided the conditions for the optics are adequate (having good signal-to-noise ratios and appropriate frequency response of the detectors, so fringes are not "dropped"), the calibration is conceptually as simple as counting the number of light and dark regions encountered by an optical detector. Complications come in tracking the direction of motion, and the massive vibration isolation tables necessary to stop detector movement with respect to the interference pattern.

The simplest counting schemes are fine for comparatively large displacements (where the stroke spans many wavelengths), but at higher frequencies, displacements are often less than a wavelength. Esoteric phenomena, such as the fringe-disappearance effect [5] for motions of precisely one wavelength, allow calibrations at certain frequencies with specially designed resonant PE shakers.

Another practical difficulty is the positioning of the mirror. The UUT is generally positioned on the centerline of the shaker. The laser measurement is desired at the surface on which the UUT should be mounted. Because the shaker can experience rotation, the displacements where the laser spot hits the mirror may be due to both acceleration and rotation. Multiple measurements are made at points around the periphery of the UUT to average out rotational effects. Alternately, the UUT can be shaken while clamped peripherally, rather than mounted normally, reflecting the laser off its exposed mounting surface. Units with large base- (or case-) strain sensitivities would perhaps have problems with this unusual technique.

Doppler-based interferometers, measuring velocity rather than displacement, are generally not used for calibration. Their detection circuitry cannot practically be calibrated absolutely.

**Primary Vibration Calibration by Reciprocity Method**

An alternate method for primary vibration calibration is the reciprocity method, using a shaker designed with two coils, one to provide motion, one to measure the motion. The shaker is a reciprocal transducer, able to drive or sense, with a "sensitivity factor" relating the ratio of drive current to force to the ratio of pickup voltage to velocity. Measurements are of absolutely known parameters: Voltage generated when the coil is driven, masses being driven, and current required. The UUT is along for the ride, and its sensitivity is determined once the accelerations are known. [14]

**Calibration by Using Earth’s Gravitation**
The most direct method to attain absolute calibration of acceleration, generally at very low frequencies, is by rotation or free fall in gravity. This requires no standard transducer or elaborate equipment, just knowledge of the local value of gravity. It can be measured very accurately and is quite stable, but the location of the measurement is important. Variations from standard gravity are as large as several tenths of a percent, depending on latitude and altitude (values decrease approaching the equator or at elevations). An approximation of the local value can be made using the
International Gravity Formula 1967,

\[ g_n = 978.03185 \left[ 1 + 0.005278895 \sin^2 \Phi + 0.000023462 \sin^4 \Phi \right] - 0.003086 \, dh \]

which takes into account latitude \( \Phi \) and altitude \( dh \), where altitude is given in meters above mean sea level. The result is given in cm/sec\(^2\). Maps of local variations can be obtained with corrections usually less than 0.05\% [15].

Rotational methods provide frequency response directly. The sensitive axis of the UUT is oriented to point up and down alternately during the rotation. The result of a constant rotation rate is a sinusoid with approximately 1 g peak amplitude and a frequency equal to the rotation rate. Static fractional g accelerations are generated by stopping the rotation at desired angles. Capacitively-coupled transducers can be given fractional g peak sinusoids by rotating while the sensitive axis is tilted toward the rotational axis.

The difficulty with PE transducers and rotational methods is getting the charge signal out through slip rings. Use of a remote charge convertor rotating with the transducer is a solution.

Particularly if rotational frequencies are high, the inertial center of mass of the transducer should coincide with the axis of rotation, so that centripetal acceleration is minimized. At constant frequency, any centripetal component will be constant (as if it were a centrifuge), with the addition of a gravitational component at the rotation frequency.

Phase response is a bit tricky to interpret if there is significant transverse sensitivity. The intent of measuring phase is to determine the time delay between the input and output sinusoids. If there is a tilt of the sensitive axis, and if the transverse component of the sensitivity is in the plane of rotation, the effective phase lag erroneously includes not only the legitimate phase delay, but also the time it takes to rotate the ideal axis through the angle of tilt to the true vertical position. (The peak output should have occurred when the sensitive axis was vertical.) This can be detected (and corrected) by rotating the transducer 180\(^\circ\) about its axis, repeating the measurement and averaging the results with the first run.

**Centrifuge Calibration**

Centrifuge testing is an absolute calibration technique generally only applicable for dc-coupled accelerometers. Capacitively coupled transducers can be tested as well, but the set-up is difficult. (It requires mounting the UUT on an additional rotator attached at a radius on the centrifuge table. As with turnover in gravity, the sensitive axis is rotated in and out of the radial direction. No more will be said of this difficult variation.)

Similar to the technique of turnover in gravity, transducer output must pass through slip rings to bring out the signal from inside the centrifuge. Low impedance output is best, so that capacitance and contact resistance variations would have negligible effect.

The input reference axis is aligned with the radius. Applied acceleration is equal to \( \omega^2 R \), in which \( \omega \) is rotation frequency (in rad/sec), and \( R \) is the radius to the c.g. of the inertial mass. Since the position of the inertial c.g. may not be known to the required precision, an additional test at a different radius can provide the additional information to calculate the actual position [16].

Moderately high accelerations can be achieved, higher than easily attained by sinusoidal techniques, but less than that using shock. In contrast to shock techniques, amplitude linearity with both positive and negative accelerations can be readily tested, by aligning the sensitive axis toward and away from the center of rotation, respectively. Rotational direction is irrelevant: Clockwise or counter-clockwise directions, corresponding to negative and positive values of \( \omega \), respectively, will give equivalent output, because velocity is squared. Sensitivity is equal (as with shock
comparison techniques) to the difference between the output during rotation and the bias, divided by the calculated input acceleration.

The stability of the bias measurement has an extreme effect on uncertainty, particularly if the UUT is prone to thermal bias shifts. Older-design PR transducers, those with discrete silicon gages, (as opposed to newer miniature designs with integral gages in a monolithic sensor), can suffer a related difficulty. This added complication may affect the sensitivity, and is a problem if the duration of the centrifuge test differs from that of the intended application. Because both resistance and sensitivity can have significant thermal coefficients in silicon gages, and because the applied force is nearly steady-state, the individual strain gages may equilibrate to different temperatures during the acceleration and have output which would differ from short-lived accelerations of the same magnitude. (Gages with lower resistance would have abnormally high self-heating due to increased current.)

Normal time durations in shock or vibration are shorter than the thermal time constants in the older gages. This should not be a factor for gages incorporated in microsensor designs, for which the thermal time constants are very short (measured in microseconds), and thermal equilibrium is maintained through nearly all types of testing.

The nature of centrifuge testing provides for a calculation of amplitude linearity that differs from that commonly encountered in vibration or shock testing. (An accelerometer with perfect amplitude linearity would have the same sensitivity, that is, ratio of output to input, at all amplitudes up to its full scale range.) With data from a centrifuge, the sensitivity can be calculated with a centrifuge using the best fit line through the output as a function of input, including the output through negative acceleration tests. The slope of this line is defined to be the sensitivity for some dc-coupled accelerometers. Higher order polynomials can be fit, and in that case the nonlinear terms can be a definition of the amplitude nonlinearity.

**Amplitude Linearity**

The topic of amplitude linearity requires further discussion, and is only briefly touched in this digression: Amplitude linearity of ac-coupled transducers, for instance, is typically characterized with shock techniques by the amplitude at which the sensitivity (based on peak comparisons) differs from the sensitivity at low levels by a given amount. For example, an accelerometer may have sensitivity at 1000 g which is 1% higher than at 100 g.

Sinusoidal testing to obtain amplitude linearity is yet another technique, often based on ac-rms measurements (in which the measurement is based on the root-mean-squared value, which sums all frequencies except the dc component) which can obscure certain types of nonlinearity. Careful interpretation of results is recommended if using sinusoidal techniques. A shift of bias due to vibration input, called vibration rectification, is a clue indicating the symmetry of the distortion, but the shape of the distortion is lost in the rms process. (For instance, a unit with nonlinearity that “flattens” the top of the sinusoid would experience a decrease in the bias measured during vibration.)

An alternate linearity technique using sinusoidal input is a Lissajous-style plot of phase-correlated UUT vs. reference transducer output, which can provide the amplitude linearity data from one period of data by the best-fit technique described above. Theoretically this could be applied to shock as well, although corrections for transient effects, such as overshoot, would be very difficult.

**Primary Shock Calibrations**

Generally two classes of absolute shock methods exist: Velocity measurements and force measurements. In the former, the UUT is given a shock and its velocity is measured by timing the travel through a predetermined distance, such as by the cutting of parallel beams of
light, or by timing the pulses of a magnetic pickup. This requires the integration of the accelerometer output to compare the output with velocity. Drag forces and rotation due to tumbling need to be estimated or controlled.

In the latter class, strain gages applied to a bat near its end responds to the strain from the force that accelerates the UUT. The strain gages are calibrated by the use of different masses.

SECONDARY VIBRATION CALIBRATIONS

This is the dominant method of accelerometer calibration for ac-coupled transducers. Comparison is made to a reference transducer on an electrodynamic shaker. Typical reference transducers use one of two “back-to-back” configurations, each designed such that the outputs of the standard and the UUT correspond as much as possible to the same surface, the surface to which the UUT is mounted.

In the first configuration, the “piggyback” or “back-to-back” design, the mounting surface is the top of the standard. At the bottom is the point of attachment to the shaker (or, in the case of shock, the anvil). When using this design, it is important to remember that the acceleration applied at the base of the standard is not what is being measured; rather, the PE sensors in the standard are physically closest to the top surface, measuring the motion of the surface by which it is connected with the UUT. An example of the back-to-back design is shown in Figure 21.

![Figure 21 Back-to-back comparison standard.](image)

In the second configuration, the assembly is a shaker armature, and the standard is buried inside, mounted upside-down under the mounting surface. Such standards generally have wider frequency response capability than back-to-back designs, particularly if the armature is made of beryllium. Beryllium has high Young’s modulus and low density, which gives it the highest speed of sound of any metal. (However, beryllium is expensive and also toxic; do not inhale machined or abraded beryllium.) This translates directly to higher values of the standard’s resonance frequency, as well as the resonances associated with the mounting of the UUT (see the discussion of boundary conditions and multiple DOE’s above). This makes possible wider bandwidth of frequency response measurements, and just as important, reduces the effect that variations in these resonances would have on the uncertainties at lower frequencies. This will be discussed further in the section on mass loading of standards.

Whichever is the standard design, the combined mass of the armature, standard, fixtures, and UUT determines the maximum acceleration level possible from the electromagnetic drive. Sinusoidal input of 10 g amplitude is common, and frequency can go up well beyond 50 kHz.

The input sinusoid needs to be very pure, having low levels of distortion. An input waveform is distorted if it includes higher harmonics, frequency components which are integer multiples of the fundamental sinusoidal frequency. In a time plot, a distorted waveform differs in shape from the pure sinusoid, depending on the amplitude and phase of the other harmonics.

Even a minimal distortion can have significant effect on lightly damped transducers. Consider a transducer with a 30 kHz resonance and a resonant amplification factor of 100, given a 10 kHz input with 0.1% third harmonic distortion. The 30 kHz component is amplified by resonance from 0.1% to a level 10% as large as the 10 kHz fundamental. In other words, although the input deviates from a pure 10 kHz sinusoid by only one part in a thousand, the output from the transducer turns out to include a 30 kHz component 10% as big as the
fundamental! This amplification can occur in either UUT or standard transducers (or to any fixture resonance), so comparisons would have to deal with such distortions on either or both channels.

This is a potent reason for use of frequency-discriminating comparison methods for sinusoidal calibrations. Such techniques, such as tracking filters, curve fitting techniques, or Fourier Transforms (whether multiple frequency FFT or single frequency correlation filtering), all serve to compare only the fundamental component of the inputs and outputs, and discard distortion components. If such discriminating techniques are not used (if rms readings are used, for instance), distortion will cause significant uncertainties, and the only recourse is to avoid taking data at frequencies near the distortion.

There are other constraints that must be considered in vibration testing. The impedance of the drive coil will limit amplitudes at very high frequencies. At intermediate frequencies the shaker is force limited. Low frequency amplitudes are displacement limited. Most small calibration shakers allow only 5 mm stroke, so amplitude of 10 g peak can be provided only at frequencies above about 30 Hz.

As described before, acceleration is proportional to the square of the frequency, so possible amplitudes for a given displacement fall off quickly at low frequencies. A one g peak sinusoid is all that is possible at 10 Hz with 5 mm stroke. Another problem at low frequency is the quality of the waveform. It is difficult to maintain linear motion through large displacements. There are many degrees of freedom of the armature on the flexure, meaning rotations and transverse motions are possible. At the extremes of travel, flexures may stretch asymmetrically, particularly if the combined e.g. of the UUT, its cable and fixtures do not lie on the centerline of the armature, or if the cable exerts transverse forces (that necessarily will vary during the stroke).

Pressurized air is often used as a non-contact hearing to supplement the transverse stiffness of the flexure. (Oil bearings are also used in "slip plates", which are designed for motion in the plane of the plate. The bulk modulus of the oil provides extremely high effective transverse stiffness, preventing motion out of the plane.) Some "long-stroke" shakers have a large area heavy plate levitated by a cushion of air. The fluid bearing is often the only transverse support as the head moves over a guide. Stroke of 150 mm is common. As the stroke increases, the ratio of axial to transverse motion can be extremely good.

Normally, amplitude limits are set by the power available in power amplifiers. However, by coupling an electrodynamic shaker to a resonant structure with low damping, large fixed-frequency amplitudes can be achieved. (In such a test, a back-to-back standard would be needed on the end of the structure with the UUT.) Some designs exist with extremely long strokes, [1 meter at 1 Hz], but inconveniently require up to a minute to build up to ultimate amplitudes [17].

Resonance amplification provides the amplitude to determining the amplitude linearity of some transducers. Usually, however, the full range of the UUT is considerably higher than can be easily attained sinusoidally, and shock techniques are required. Shocks often better simulate field conditions than sinusoidal testing. Pure sinusoids are uncommon in nature.

**Secondary Shock Calibration**

Many shock exciters use a fixed amount of momentum, usually determined by the height of a free fall, and a variable amount of padding at the point of contact. The thickness and stiffness of the padding, sometimes called a programmer, determines the interaction time and therefore the force. The combined mass of the anvil, the standard, and the UUT determines the acceleration. Figure 22 illustrates why it is useful to be able to control the amount of momentum.
The two shocks shown have the same impulse (the hatched areas are equal). Interaction time and displacement determine amplitude and pulse duration. Although the momentum change is equal for the two events, the displacement traveled during the short, hard impact is much smaller than the long, cushioned pulse. To attempt to increase duration without changing amplitude (or vice versa) requires an increase in momentum change.

As with sinusoidal calibrations, most shock calibrations are based on comparison to a standard. There is a significant difference between the techniques, however. Sinusoidal calibration uses an input waveform which has been carefully crafted to be single frequency, and/or the frequency of interest is extracted by filtering or other discrimination techniques. In shock calibrations, by contrast, sensitivity of the UUT is commonly established by comparing the peak output values of the time-varying UUT and standard channels. Shock comparisons are done in the time domain. This has many difficulties.

The comparison method during shock is made more complicated by the wide frequency content of a shock waveform. Rather than the single frequency of a sinusoid, the waveform of a transient event is literally made up of an infinite number of frequencies. Shock comparison signal conditioning commonly passes both inputs through equivalent filters. The intent is to take out tilt: frequency components which are likely to differ between the two channels. The shock period is made to be neither too long nor too short, and the shape of the pulse is designed to be smooth, so the dominant frequencies are within the passband of both transducers.

Historically, the rule of thumb is to keep the duration of the shock at least five times longer than the natural period of the UUT (although the dominant resonance of the anvil/standard/UUT stack is the more appropriate criterion). If the shape of the pulse deviates from a half-sine, (tending inure to that of a sawtooth), the frequency content is considerably higher, and slower rise times should be used. Short pulses on underdamped accelerometers result in overshoot. When time-domain peak comparisons are used, the overshoot can be misinterpreted as a higher sensitivity due to an amplitude nonlinearity. It is difficult to tell the difference between the effects of frequency response and amplitude nonlinearities.

In sinusoidal calibrations, generally the comparison is made one frequency at a time. Some contrasting systems use “random” vibration calibrations which use a technique that is more applicable to shock. Specifically, comparison is made not in the time domain, but in the frequency domain. The input is an approximation of “white noise” vibration, a type of waveform in which, over time, the average contribution of each frequency component is uniform (that is, the spectral amplitude between 10 and 11 Hz is the same as between 100 and 101 Hz or 10 000 and 10 001 Hz, etc.). Throughout the random test, a wide band of frequencies make up the input and multiple spectral analyses are performed. The analysis uses a mathematical algorithm which separates the waveform into its constituent components, averages many such spectra over time, and compares the plots for the two channels.

Random vibration testing has the luxury of relatively long duration input, to allow a wide band of frequencies and a statistically significant number of measurements of the response of both accelerometers to the input frequency components. Shock testing could use the same frequency analysis techniques, although accuracy may not be as good. The practical difficulty along pulse durations (more momentum, mechanical energy, and forces) and the fact that the shock may be difficult or impractical to repeat, (disallowing the benefits of averaging) means signal-to-noise ratios for shock will be smaller than those of vibration testing.

Another difficulty in shock calibration is the absence of direct traceability to national standards. Traceability
is commonly established by extrapolating from low level acceleration measurements of the standard accelerometer using its amplitude linearity. Amplitude linearity of Endevco’s model 2270 standard, for instance, was established by simulating shock loads on the sensor elements by using large masses and lower accelerations [18].

Probably the biggest limitation with comparison shock calibrations occurs when pulse durations are below approximately 100 microseconds, which is common when shock levels above 10 000 g are attempted. Any back-to-back technique relies on rigid-body motion of the UUT and standard. The finite propagation velocity of a pulse through any material guarantees there is relative motion between the UUT and standard. The correlation between the motion of the two transducers breaks down, while reflections and transverse stress waves in the gelatinous goo of the anvil/standard/UUT may make the comparison invalid. Such complex motion tends to occur for high frequency components whose wavelength in the material are comparable to the dimensions of the structure.

There are contrasting shock techniques in which the intent is to make one dimension of the calibration structure large relative to the wavelength of the pulse. The pulse is embodied as a compression strain wave in a long slender elastic bar (called a Hopkinson bar) which serves as a mechanical waveguide. One technique of calibration compares the strain in the bar (measured with strain gages which serve as the standard) and the velocity experienced by the test transducer [19]. Other techniques determine acceleration levels through differentiation of the strain output, after making corrections for the frequency characteristics of the bar and effect of the mass of the accelerometer at the end [20, 21].

Like the back-to-back comparisons, these techniques also have limitations related to the pulse duration. The low frequency limit is based on the length of the bar; the high frequency limit is set by the diameter. However, because strain is the standard measurement, the extrapolation required of the amplitude linearity of the standard is less extreme than in the back-to-back comparison techniques. Instead, different amplitudes of accelerations involve differing strain rates. The frequency response of the gages and signal conditioners determines the uncertainties. The momentum levels achievable permit calibrations even up to 200 000 g with reasonable rise times and strain levels.

For all shock techniques, intimate contact between the UUT and the mounting surface is vital, not only for calibration accuracy, but also for UUT survivability. Shocks can involve large negative accelerations, and if the UUT is not well mounted, parts of it can momentarily leave the surface and slap back down with often disastrous results.

Symmetry and alignment are also critical. The impact position and direction must be in the sensitive axis and through the inertial e.g. of the UUT and the anvil/standard/UUT assembly. If the impact is off-axis, transient rotation is the result, and not only may transverse sensitivities come into play, the rotational and centripetal accelerations may add to the axial acceleration at the UUT.

**Transverse Vibration Sensitivity Testing**

A complete measure of output in the sensitive and transverse axes would require tests in three independent directions. If one direction is perpendicular to the mounting surface, the other two, if orthogonal, would define the surface itself. The transverse sensitivity, if any, lies in that plane. Remember from the discussion above that 1) a tilt in the sensitive axis away from the ideal orientation creates the transverse sensitivity, 2) there is a direction of maximum transverse sensitivity in the plane, and that 3) 90° to that direction in the plane, sensitivity is zero. With measurements in three orthogonal directions, each of these directions can be determined. The maximum transverse sensitivity would be the square root of the sum of the squares of the outputs at the two transverse directions. The difficulty
is in aligning the shaker motion with the mounting surface of the transducer. Accelerations which deviate from the plane would excite the sensitive axis, and the measurement of the small transverse component would be dominated by the output from that much larger sensitive axis.

Precision of the UUT mounting alignment, in addition to purity of the direction of input acceleration, are therefore key to any transverse sensitivity measurement. The input acceleration must be aligned with the mounting surface throughout the stroke. The ideal accelerometer has zero output with transverse sensitivity, so the intent is to measure that nothingness with certainty. If a precision of 0.1% (of the sensitive axis value) is desired, trigonometry dictates that the input alignment needs to be better than 1 milliradian, and the cross axis motion of the shaker (which in this case would be in the sensitive axis) would need to be less than 0.1% of the primary motion. This is an extraordinary requirement, generally accomplished only with large displacement shakers with stiff bearings to control off-axis motion. Because the value to be measured is small, large accelerations are needed for signal-to-noise ratio. The fixturing required to align the UUT in various orientations are often large enough that the forces and torques in the shaker overcome the bearings, and motion degrades. Fixturing must be compact.

Common electrodynamic shakers generally cannot be used for transverse testing. Specifications for transverse motion even of high quality calibration shakers are still on the order of 3% from 100 Hz to a few kilohertz, and considerably more at flexure resonance frequencies, although such motion has little effect in normal sensitivity measurement.

As an example, for a UUT with 1% transverse sensitivity, and 3% transverse motion of the shaker, maximum contribution of the transverse output to the primary sensitivity measurement would be the product of the two, only 0.03%. However, using such a shaker to evaluate the transverse sensitivity in the example would be futile, resulting in apparent sensitivity on the order of 3%, rather than the correct 1% actual value.

**Base Strain Sensitivity Testing**

Base strain sensitivity is the output from an accelerometer caused by strains at the mounting surface. Often in the field (or occasionally on a shaker in a laboratory), the events cause simultaneous acceleration and bending of the structure. Any base strain sensitivity can result in output that is difficult to distinguish from the acceleration signal. In calibration, this could cause a change of apparent acceleration sensitivity. This might be dependent on mounting torque value used, and is likely a response specific to that mounting configuration.

Correction for known strains is unlikely, even if the nature of the strain were known. Output is often nonlinear, and different types of strain, such as pure tension, shear or bending due to curvature, can affect the transducer differently. If the structure under an accelerometer is expected to bend or strain during a measurement, the accelerometer should be selected for minimum strain sensitivity, or strain isolation techniques, such as the use of an isolating mounting stud, should be considered.

In the interest of standardization (for comparison of accelerometer designs), the American National Standards Institute recommends a test with 250 microstrain, developed on a beam with radius of
curvature of 25.4 m [1000 inches] [22] Sensitivity is expressed in equivalent g per microstrain, although experience has shown that the output is not linear and the ratio should not be extrapolated. Since the test involves oscillations of the beam [so that ac-coupled transducers can be tested] correction needs to be made for the actual accelerations imparted during test.

In dc-coupled transducers, another form of strain sensitivity may be apparent as a shift in ZMO when torque is applied to mounting screws. Sometimes referred to as torque sensitivity, this related phenomenon can be mistaken for shock-induced zero shift if the event causes mounting screws to lose preload.

The strain sensitivity of an accelerometer can be significantly reduced by mounting it on an insulated stud or a stud with a thick flange. This will also cause a reduction in the mounted resonance frequency. If such a fixture is to be used in the application, the accelerometer should be calibrated with it to ensure that its true mounted performance is obtained.

**Thermal Testing**

Thermal testing is done to measure the effect that temperature has on performance of a UUT. Although in a strict sense, even “ambient” tests are thermal tests [see next section], the term refers to conditions requiring special equipment, which can allow practical tests ranging from near absolute zero to above 650°C. The most common parameters tested are bias and sensitivity; frequency response can be more difficult.

When absolute techniques cannot be used, such as gravity, comparison techniques must be used. This raises the problem of control of the temperature of the standard, which usually needs to be outside the environment, connected to the UUT by a “stinger”, a rod that should be both thermally isolating and mechanically transmissive. The UUT is normally in a forced-convection environmental chamber. High velocity flow is used to maximize the heat transfer of the air, since the stinger can represent a significant thermal drain and have a large thermal gradient near the UUT. It is best to take a temperature measurement at the UUT [and not a bad idea to measure at the standard outside, as well], with the probe physically in contact with the mounting fixture. It is easy to be fooled by the air temperature reading.

The method must be used at frequencies low compared to the longitudinal [or other] resonance of the standard/rod/UUT assembly. Depending on mass and stiffness configurations, the dynamics can be quite complicated, with many possible DOE’s [such as bending or twisting]. In general, search for a frequency band without resonances, and pick a test frequency in the middle of the band. Frequency response is seldom tested with this configuration.

With very moderate temperatures, it is possible to test frequency response with the back-to-back standard in the chamber, (of course this is NOT recommended practice for a “golden” laboratory standard). Although the absolute sensitivity may be uncertain, the frequency response of the standard is easily shown to be little affected by temperature, and the response of the UUT, being a ratio, is more valid.

**UNCERTAINTY IN MEASUREMENTS**

First, uncertainty and related terms are defined.

The calibration process begins with the concept that there exists a value of a measurand, for example the sensitivity of a transducer, which is desired to be known at some required accuracy. A specification of the measurand is required [such as defining the frequency, amplitude, and temperature, with appropriate tolerances for each], to a degree of exactness corresponding to the required accuracy.

A measurement is made with a method and a procedure, producing a result which is, at best, an estimate of the ideal value. Reporting the result of a measurement is complete only by including the uncertainty, which is the
doubt about the exactness or accuracy of the result of that measurement. The uncertainty will be an estimate itself, given as an interval around the result, within which the measurand may be expected to lie with a stated level of confidence. [Quantification of uncertainty and confidence levels is described in the section Estimation of Measurement Uncertainty.]

Relating to these definitions, uncertainty in measurements in general, not just calibration, comes from many sources [23]:

a) Incomplete definition of the measurand. The failure of the definition may result from sensitivities of the test devices to be calibrated.

b) Imperfect realization of the measurand definition.

c) The sample measured may not represent the defined measurand.

d) Inadequate knowledge of the effects of the environmental conditions on the measurement procedure or imperfect measurement of the environmental conditions.

e) Personal bias in reading analog or digital instruments.

f) Instrument resolution or discrimination threshold.

g) Values assigned to standards and reference materials.

h) Values of constants and other parameters from external sources and used in data reduction.

i) Approximations and assumptions incorporated.

j) Variations in repeated observations of the measurand under apparently identical conditions.

**SOURCES OF UNCERTAINTY IN ACCELEROMETER CALIBRATIONS**

Elaboration is now given of the entries in the table titled “Do’s and Don’t’s in Accelerometer Calibration” given at the beginning of the paper. The specific topics are temperature, relative motion, mass loading, ground loops (and shielding), transverse motion, mounting techniques, cables, handling, and signal conditioning. Nearly all require study and understanding by the operator and the designer of procedures and systems. Outside of the scope of the discussion is perhaps the most perplexing problem in the laboratory: How to reduce operator-induced variability.

Good design of procedures, training, and to some extent the degree of automation in the calibration equipment, may help. However, the bulk of the sources of uncertainty can be directly affected by the execution by the operator. Contamination of mounting surfaces, improper mounting techniques, damage by mishandling, inappropriate grounding schemes, etc. are examples.

Other “obvious” problems, such as noisy cables, intermittent electrical connections, broken shaker flexures, blown fuses, failed voltage regulators, etc. will also be ignored in this discussion, despite the fact that they can represent, until they are uncovered, some of the most puzzling sources of variability.

Other, subtle sources of uncertainty, which will not be discussed, include the effect of the history of the UUT and the calibration equipment. For instance, degree of zero shift in PE accelerometers can be related to the time elapsed since the last shock.

Because a calibration is dependent on so many parameters, such as amplitude, frequency and temperature, these conditions are to be reported with the results of a calibration. This should be true of any measurement; all pertinent variables [the trick is knowing which are pertinent] should be specified so subsequent calibrations can duplicate the conditions. Also, it is important to note the dimensions and mode in which the respective parameters are expressed [e.g. average, rms, or peak]. For accelerometers, peak readings are assumed unless otherwise noted. It is best to use the same term for both numerator and denominator [e.g. mVpk/ms^-2pk, or pCrms/grms] rather than mix terms. The relationships between average, peak and rms are different for different waveforms [1].

**Temperature**

The temperatures of the laboratory can affect most parameters of a calibration, such as the sensitivities
of both the reference and test accelerometers, their capacitances, bias voltages, impedances, damping characteristics (oil damped units), gain of signal conditioning, and so on. An estimate needs to be made of the effect of these temperatures on the calibration uncertainty, along with a record of the temperature(s) during test, and what corrections were made (if any) to compensate for the known thermal characteristics.

Corrections could be made for thermal effects on any components in the system for which the thermal performance is known. (Corrections are not made for the UUT, instead the test temperature is simply given with the calibration). However, it is more common to establish a maximum uncertainty due to the expected thermal excursions. Variations of laboratory conditions during the day, such as those accompanying changing air conditioning loads, must be determined to provide an accurate estimate of these uncertainties in the accelerometer calibration.

Where the temperature is measured is important. Temperatures inside the console enclosure, where temperature-sensitive amplifiers may be, may differ from ambient. A shaker armature temperature may differ significantly from ambient air around the console, particularly due to power dissipation during large amplitude tests at high frequencies.

When the temperature measurement is made is also important to the overall calibration uncertainty. Generally measurements should be made after temperatures have stabilized. The required delay will depend on the thermal differences, thermal time constants, thermal coefficients of the parameters involved, and the desired accuracy. Laboratory enclosures may have a long thermal time constant, with warm-up times approaching an hour. Several minutes delay may be needed after mounting a transducer, if the temperature of a mounting surface differs from the UUT’s storage temperature. Thermal transients can be significant for other reasons, such as the effect of large pyroelectric output on signal conditioning.

Relative Motion and Mass Loading
To review, it is assumed that in comparison vibration calibration both the reference accelerometer and the UUT experience the same acceleration. Because their respective inertial centers of gravity are separated in space by material, however, there may be relative motion between the sensors of the two accelerometers, resulting in phase shift and possibly a change in the ratio of motion between them.

The mass of the UUT will load the case stiffness of the back-to-back reference, changing the relative motion.

Relative motion, if repeatable, can be characterized and corrected. The correction will have uncertainty, associated with the non-repeatability. The magnitude of both the motion and the uncertainty becomes larger as the resonance is approached. The primary reason why it is advised to use a standard with the highest resonance frequency (as measured with UUT attached) is that effect of the variations of the resonance will be lower at normal calibration frequencies.

Figure 24  Mass loading/relative motion correction factors for Endevco model 2270. The effects of mass loading by the UUT are generally reduced if the mass of the standard is large compared to the UUT, which is why correction factors are generally smaller for standards built into shaker armatures.

Fixtures add more material, and will change the stiffness between the UUT and the armature surface, making the relative motion larger. The softer the overall spring constant, the lower the resonance frequency of the standard/fixture/UUT system and the larger the relative motion. (Of course mass, too, controls the resonance. Its effect is discussed separately in a later section).

Relative motion, if repeatable, can be characterized and corrected. The correction will have uncertainty, associated with the non-repeatability. The magnitude of both the motion and the uncertainty becomes larger as the resonance is approached. The primary reason why it is advised to use a standard with the highest resonance frequency (as measured with UUT attached) is that effect of the variations of the resonance will be lower at normal calibration frequencies.
The effect of relative motion may be observed in the examples in reference [25] where mounting adaptors of identical dimensions but different materials were used. In one test with a small UUT, relative motion of greater than 10% were observed at 20 kHz when the steel or aluminum fixture was used. Using a beryllium fixture, the data closely approached the results with the accelerometer mounted directly to the shaker. A heavier UUT may have experienced greater relative motion.

The stiffness between the reference accelerometer and UUT is also a function of the mounting surface coupling area. If the surface mounting area of the UUT is reduced, stiffness is reduced. Similarly, if the UUT base surface is not completely seated on the mounting table, for example when the UUT is larger than the back-to-back reference transducer, larger deviations will again occur.

Where a conventional back-to-back reference standard does not provide sufficient mounting area, it is advisable to use an armature with integral standard. Alternately, beryllium-cased back-to-back standard accelerometers exist, which are effectively the same as a integral standard shaker, but which can be mounted on a more conventional shaker. In addition, they can incorporate mounting thread patterns for direct mounting contact to the vibrator table of the UUT base for various thread sizes and pitches, eliminating the need for adaptor fixtures.

Mass loading can affect measurement uncertainty due to its effect on its strain sensitivity. The back-to-back standards (or a shaker armature with integral accelerometer), must bear the inertial forces of accelerating the UUT. If strain isolation in the design of the standard is not adequate, some of these stresses appear at the sensing elements in the standard. The intent is to sense only the stresses from accelerating the inertial mass of the standard, not the variable stresses from the mass of the UUT.

A particularly important system consideration is prevention of ground loops [26]. The problem will occur when the common connection (or signal return) in the system is grounded at more than one point. Differences in earth potential up to several volts may exist between various grounding points. This potential difference can produce circulating ground currents that introduce noise in the measuring system.

Ground loop problems caused at power line frequencies are a phenomenon well known as a cause of measurement uncertainty. Another source, perhaps not as often acknowledged, comes from the power amplifier output. The driving coil is capacitively coupled to the armature of the vibrator. Therefore, a potential exists at the armature table. When the reference and UUT accelerometers both have their output low sides common with their housing mounting bases, a ground loop can only be prevented by keeping the input ground of both signal conditioners separated.

If a ground loop is present, the signal coupled from the power amplifier can add to the transducer output. That component is at the same frequency as the acceleration signal, so cannot be distinguished, and it normally increases as the frequency increases due to the decrease in capacitive impedance. This signal will decrease as the shaker approaches its resonance, because less voltage is required from the power amplifier near resonance to produce the required acceleration level. A dip in apparent response near a known armature resonance would be a strong clue that this possible source of uncertainty is a problem.

The most severe effects of such a ground loop component are observed when the UUT has a very low sensitivity. With an accelerometer with 0.8 pC/g sensitivity in a test shown in reference [25], an error of about +3% at 10 kHz was observed for data obtained when a ground loop was present.

It is important to observe correct grounding to ensure accurate calibration. The only method of preventing
ground loops is to ensure that the entire system is grounded at a single point. In general, the most satisfactory system ground point is at the readout input. (When several channels of data are being simultaneously fed to the same recorder, this is mandatory.) This requires that both accelerometer and amplifier outputs be insulated from ground.

One technique of accelerometer isolation involves electrically insulating the sensing element from the transducer housing to provide a floating output. With this approach the accelerometer case is at ground potential, but is not connected to the “low side” of the signal. This method, however, has a serious drawback. Capacitive coupling between case and transducer element permits coupling of AC noise directly into the (high impedance) transducer.

It is better to isolate a normal case-grounded accelerometer electrically from the structure to which it is mounted. Using this method, while the accelerometer is removed from earth ground, the transducer element is still shielded by the transducer case at circuit ground potential. Good mechanical coupling is still needed to maintain frequency response, however, and if the isolation is necessary in field application, calibration should be performed with the stud. Insulating mounting studs can provide stiff but temporary electrical isolation, and are the recommended technique for transducers with replaceable studs.

Alternately, the transducer can be adhesively mounted with a non-conducting adhesive [see section on Mounting Techniques, below]. However, any surface imperfections may short out the isolation. Isolation can be ensured by “sandwiching” an insulating material between the transducer and the structure.

Some automated calibration systems perform a check of grounding conditions to set the isolation condition of the signal conditioners [25]. Some back-to-back standards allow a manual choice of isolation schemes, using a grounding nut on the connector, the position of which selects the whether the case of the UUT and the standard are common or isolated.

If the chassis of the signal conditioning is tied to circuit ground, isolation from the console frame can be achieved satisfactorily by wrapping with insulating material (electrical tape, etc.) or by simply placing on paper or cardboard. If amplifier output cables are unjacketed, care must be taken that any exposed shields or connectors do not become inadvertently grounded ahead of the calibration system input.

Transverse Motion
Transverse and rotational motions generated by the shaker result in increased measurement uncertainty. The UUT and the reference accelerometer are unlikely to have the same transverse sensitivity and orientation relative to the transverse motion, nor will they even experience the same motion. With any rotation, the two will experience different accelerations because they are not at the same radius relative to the armature c.g. Therefore, to minimize measurement uncertainty, the shaker must maintain low transverse motion over the calibration frequency range.

Typically an armature will exhibit transverse resonances at several frequencies. Because transverse stiffness is generally much smaller than axial stiffness, measurement uncertainty caused by transverse motion can be minimized by using a calibration shaker with high axial resonance frequency. An example of this is given in reference [25].

Mounting Techniques
Transducer mounting techniques and surface conditions can also affect the uncertainty of the measurement and the repeatability of calibrations, particularly at high frequencies.

The best method is to mount the UUT with the appropriate adaptor studs or bushings, so that the entire base of the UUT is in good contact with the surface of the reference standard.
These studs are of the correct length and incorporate a flange to prevent "bottoming" of the screw in the accelerometers.

Care should be taken to ensure a flush mating with a smooth, flat surface. Nicks, scratches, or other deformation of the mounting surface on either the UUT or the reference transducer will increase transverse sensitivity and degrade frequency response. Inspect the accelerometer before using it. It is recommended that the mounting surfaces and tapped holes conform to the following specifications. These are considered to be easily achieved by following good machine shop practices:

<table>
<thead>
<tr>
<th>Specification</th>
<th>Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface flatness</td>
<td>0.0003” TIR</td>
</tr>
<tr>
<td>Surface roughness</td>
<td>32√</td>
</tr>
<tr>
<td>Perpendicularity of hole</td>
<td>±1°</td>
</tr>
<tr>
<td>Tap class</td>
<td>2</td>
</tr>
</tbody>
</table>

A thin application of a light lubricant or acoustic couplant will improve transmissibility, filling voids with nearly incompressible fluid and thereby increasing compressive stiffness of the joint. This is particularly important for calibration above 5 kHz, at which any changes in resonance have significant effect on measurements. If an insulated mounting stud is being used, make sure that the lubricant used is nonconductive. In any case, safe mounting practices dictate that couplants should not be allowed to creep across the insulating layer.

A torque wrench should be used to mount all accelerometers to ensure repeatability in the installation of the transducers and to prevent damage by overtorquing. The mounting torque recommended by the manufacturers of the reference standard and UUT should always be used.

Because of their effect on frequency response, insulated mounting studs, cementing studs or mounting fixtures are to be avoided whenever possible, (in general, any extra interface), tailless they duplicate the field mounting condition. When required, however, take advantage of the opportunity to design the fixture to minimize another possible calibration uncertainty: Off-axis alignment of the inertial c.g. of an accelerometer. If it is not centered on its mounting hole pattern, a fixture which offsets the mount to align the c.g appropriately will improve calibration uncertainties due to any rotation present in shaker motion.

Calibration of triaxial accelerometers without the correct fixtures can be particularly troublesome for the same reason. Good fixture design orients the accelerometers such that the inertial c.g. is coincident with the centerline through the shaker armature and reference accelerometer. Keep in mind that the total c.g. of the armature/fixture/UUT should also be maintained at the centerline, if possible, to reduce creation of transverse motion.

An alternate mounting technique, using cyanoacrylate rapid bonding adhesives, is quite effective for calibration when properly applied [27], but they are not recommended for applications other than ambient temperature conditions and moderate acceleration amplitudes. The bond joint is very thin, and, if area is large enough, provides fair strength and mechanical transmissibility.

Dismounting an adhesively-applied transducer deserves comment. They should not be removed with impacts, but instead with solvents, allowing softening of the bond, supplemented by light shearing torque. All traces of adhesives should be removed using recommended solvents only. A razor should NEVER be used, risking nicks, scratches and other damage to the critical mounting surface.

**Cables**
Connectors and cables are often the weak link in an instrumentation chain. Noise and outright measurement failure can be caused by intermittent connections (such as worn contacts, contamination or corrosion), or in many circumstances cables have been known to
generate more output than the transducer. Worse, often
the signals from cables are not suspiciously large, are
correlated in time with the acceleration measured, and
thus are very difficult to distinguish by inexperienced
operators.

When coaxial cable is flexed, squeezed, or in some other
way mechanically distorted, the shield and dielectric
may separate. The result is a generation of triboelectric
charges, depicted in Figure 25. The charges on the
dielectric are trapped by the low conductivity of the
dielectric material. The charges on the shield, however,
are mobile and travel from the inner conductor through
the terminating impedance, generally the input stage
of the coupling amplifier. This momentary current flow
produces a signal pulse at the amplifier input. When
the cable distortion is relieved, dielectric and shield
are joined together and the formerly trapped electrons
now flow into the shield, resulting in a second pulse of
opposite polarity.

![Figure 25 Triboelectric effect in coaxial cables. A cross-section is shown, depicting a separation of conductor and an insulator layers due to a bend radius that is too small. The mobile charges in the shield can travel through an external conduction path (the charge amplifier), while the charges in the dielectric are immobile. The direction of current reverses when the layers return to their original position.](image)

Noise-treated cables have a conductive coating applied
to the surface of the dielectric which prevents buildup
of local charges during mechanical separation from
the shield. This treatment greatly reduces triboelectric
noise. Noise-treated cable should always be used with a
PE accelerometer.

Shielding and cable placement contribute to cable-
induced uncertainties. In high impedance PE circuitry,
improperly shielded cables act as antennae and pick up
charge from voltage fields. Magnetic fields are picked
up by neatly coiled cable loops, and can cause common
mode signals (on input and shield lines) which may not
be handled well by the signal conditioning. Long cables
can change system frequency response, by loading
the transducer output and amplifier input. Remote
charge convertors are recommended in extremely noisy
environments, and when very long lines need to be
driven. (Noise-treated cable is still needed between the
transducer and the charge convertor, but less expensive
conventional cabling can be used downstream).

PR transducers have related uncertainties due to long
cables. Capacitance between conductors acts as a low
pass filter, and series resistance of the input conductors
dissipates excitation voltage. When fatigued and
strained, the cable conductors may also become strain
gages themselves, varying resistance as they move.
(This is generally a problem only for conductors carrying
excitation current. For instance, “quarter bridges,” with
only one gage at the end of the cable, can have this
problem.)

Mechanical characteristics are also important.
Cable weight and stiffness affect the UUT, reference
transducer and the shaker motion. Good transducer
cables are as small, light, and flexible as possible,
although some applications demand rugged mechanical
protection. Such stiff or massive cables can severely
distort normal response, particularly with lightweight
UUTs and calibration shakers designed for high
frequency. Shakers with very stiff flexures are needed,
which typically have armatures with low axial resonance
frequencies and large transverse motion at frequencies
above 3 to 4 kHz. Calibrations of these accelerometers
will have high measurement uncertainties at higher
frequencies.

Most cables are comparatively fragile. Cables should
not be stepped on, kinked, or given excessive clamping
forces. When attaching (and detaching) cables, care must be taken not to bend the center pin of the cable connector, which is relatively susceptible during these operations. The connector should be kept clean and dry. Low insulation resistance due to moisture can cause noise and apparent anomalies in frequency response. (This is true for the transducers as well. Occasionally the environmental seal of a transducer fails, allowing internal moisture to degrade electrical isolation. The transducer generally can be recovered with a bakeout in a vacuum oven.)

Torque on the cable connector can be important. Sonic transducers respond to cable-induced forces, and frequency response plots can show connector resonances which vary considerably with torque. Generally, “finger tight” is adequate (despite having widely varying actual values) for miniature connectors on properly designed transducers. Tools should not be used on connectors which with knurled, roughened surfaces designed for fingers. However, some connectors are hex shaped, [such as on the smallest transducers] and should be used with the proper wrench and recommended torque settings to prevent damage to the transducer.

Mechanical cable effects on response are usually caused by improper strain relief techniques. When possible, the cable should be clamped, within 10 to 20 times the cable diameter of the transducer connection, to a surface which is experiencing the same motion as the transducer. This prevents relative motion and strain. Bend radii of less than a few radii should be avoided; they overstress materials and cause separation of insulators and conductors. The contact to the clamp should be spread over a length of at least 5 diameters to avoid damaging the internal insulation. The length of the cable strain relief loop is a compromise: If too short the motion may induce axial forces; If too long, cable whip can lead to cable fatigue and strain effects in the transducer.

If a cable is suspected of having damage, it is best to discard and replace it. Loss of data, or corruption of calibration or test accuracy can be far more expensive. The repair of any miniature coaxial cables is not recommended, particularly any noise treated designs. It is difficult to make a reliable noise-free repair.

Accelerometer Handling

Accelerometers are generally rugged, but anything can be damaged. Typical accelerometers dropped from a height of 3 feet to a typical laboratory floor, wood or asphalt tile, are subjected to a shock amplitude of 3000 to 5000 g. Overshoot in an underdamped transducer can amplify the effect. Metal-to-metal contact can generate much higher amplitude and shorter durations shock pulses. Flat-to-flat metal contact can be devastating; it is not difficult to generate several hundred thousand g’s with contact between smooth well aligned surfaces.

Signal conditioning and electronics

When PE accelerometers are used with charge amplifiers, the system low frequency response is determined primarily by the amplifier. Amplitudes are attenuated by the ratio

\[
\frac{f}{f_c} \left(1 + \frac{f}{f_c}\right)
\]

and phase angles are given by \(\tan \Phi = f/f_t\), where \(f_t\) is the corner frequency of the high pass filter.

The frequency response of the signal conditioner at different gain settings can be a source of measurement uncertainty. The specified limits for typical signal conditioners for gains versus frequency at all gain settings are generally not adequate to meet the desired low measurement uncertainties of a good calibration system. Fortunately a major component of these uncertainties can be eliminated by calibration of the system.

Some automated calibrations systems allow an end-to-end electrical comparison calibration, periodically performed over the entire calibration frequency range for each input channel at each gain range. The electrical
calibration results can be applied to each UUT vibration calibration to minimize measurement uncertainty. It is assumed that short term variations in frequency response characteristics are not significant. To correct for short term random variability in amplifier gain, the system gain ratio may be electrically calibrated each time a UUT is calibrated. The user-selected reference frequency would be used. With the use of an automated computer controlled calibration system it is possible to perform this calibration rapidly on a routine basis. It is important to use the signal conditioning near its full scale for maximum signal-to-noise ratio. In some calibrations such as frequency response sweeps, the output of the UUT changes significantly, so gain settings should be adjusted as appropriate during the course of the calibration.

**Magnetic Sensitivity**
Generally, shakers designed for higher frequencies tend to have larger magnetic fields at the mounting surface. The effect of these fields is a function of the accelerometer design. Some transducers (generally old PR designs) include masses made of ferromagnetic materials, in which magnetic forces can be induced. These forces have periodic components which are generated as the masses pass through gradients in the magnetic fields, and thus are indistinguishable from the sinusoidal acceleration. The larger the gradient in the magnetic field, the larger the magnetic effects. Mounting the accelerometer on a stinger, a long rod which simply places the transducer far from the shaker and the fields, will reduce the effect. As with thermal testing, the method must be used at frequencies low compared to the longitudinal resonance of the rod. It should be noted that the presence of the stinger may increase the transverse motion of the shaker head, particularly if it is cantilevered horizontally in gravity.

**ESTIMATION OF MEASUREMENT UNCERTAINTY**
Several techniques have been developed for the calculation of the uncertainty of accelerometer calibration. Two are most widely used, but a third is now recommended [23]. Differences between the techniques are subtle, but they affect how the estimates are interpreted and how easily they can be transferred when using the results in subsequent uncertainty analyses. The techniques will be briefly described after a [very] brief introduction on statistics. The key terms here are defined following the sequence taken when calculating measurement uncertainty. [An excellent discussion of these topics is given in Reference 23.]

### BACKGROUND: STATISTICS
A series of independent measurements of a process taken under the same conditions will have variability, with each reading having dispersion, \( |\mu - x| \) which is the difference between its value and the mean \( \mu \), the average value.

\[
\mu = \frac{1}{n} \sum_{i=1}^{n} x_i
\]

The sum of the squares of the dispersion for the readings, divided by one less than the number of readings, \( n \), is the variance.

\[
\sigma^2 = v = \frac{1}{n-1} \sum_{i=1}^{n} (\mu - x_i)^2
\]

Divide \( \sigma \) by the square root of \( n \), and the result is a smaller number, the standard deviation of the mean, also known as the standard uncertainty. It is important to distinguish carefully between the standard deviation of a sample, and the standard deviation of the mean. These terms are illustrated in the following examples.

\[
\sigma_m = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n} (\mu - x_i)^2}
\]
rectangular (values have equal probability of occurring within an interval) or normal [the familiar bell-shaped curve of Figure 26].

![Figure 26 Normal distribution](image)

Figure 26 Normal distribution. In the case of normal distribution, the most common values are near the average or expected value \( \mu \), at the hump in the middle, and values which are less likely are farther out on the "tails". The percentages shown are the fraction of values which lie within an interval about the average, with a span defined by a number of standard deviations.

The reason the normal distribution is so common is described by the Central Limit Theorem, which states that combining the result of many parameters results in an approximately normal distribution, no matter what type of distribution each parameter has, provided that the overall variance is much larger than the variance of any one non-normal component. This enables the use of simplifying assumptions and relationships in many problems dealing with unknown probabilities.

Consider a distribution of a large number of test results, (each test in the distribution must be taken under the same conditions, of course), each test result being the average of readings of some arbitrary distribution. Because of the number, no one result will dominate the variance of the set, so the overall distribution is normal. With a very large number of tests, the mean will approach the expected value, and it can be predicted that 68% of the values will fall within an interval \( \pm 1 \sigma \) on either side of the mean.

With a smaller number of samples, the mean will be just a guess of the value being measured. Because the number of samples is finite, it turns out that the mean itself is a random variable, taking on different values depending on the chance occurrences of the tests. The nature of the distribution of the mean is characterized by the standard uncertainty. To illustrate, if the tests were repeated many times, and the sequence of means analyzed, (imagine many separate sets of tests, each with its own mean), the values of the means would also have a normal distribution, with its own mean, [the mean of the mean]. This distribution of mean values would also have its own standard deviation of the mean (hence the name) defining the interval in which 68% of the means would lie. This is the value more simply approximated by dividing the standard deviation of the original sample by \( \sqrt{n} \).

For a numerical example showing the utility of multiple measurements, a measurand might have a significant random component, with a \( \sigma \) 10% as large as the mean. By averaging 100 samples, the standard uncertainty due to this factor would be \( \sqrt{n} \) smaller, only 1%.

To continue with standard definitions, there will be many influence factors in the measurement, each with its own standard uncertainty and each with an effect on the result, some more critical than others. By combining the effects of the factors, according to the mathematical model of how the factors relate to the overall result, a combined uncertainty is achieved. (For instance, a sensitivity is calculated from several terms, such as voltmeter readings, gains of amplifiers and the sensitivity of the reference transducer, and is affected by mass, transverse motion and temperature, among many others. Variations in temperature would probably have much less direct effect of the result as would variation in gain, for example, so when combining the variances of each of these parameters, the weighting coefficient for the variance of temperature would be smaller.)

The final definition in this introduction is the confidence level for the uncertainty estimate. Multiplying the combined uncertainty by a coverage or \( k \) factor, typically in the range of 2 to 3, gives the expanded uncertainty, (sometimes called the overall uncertainty). The chosen values of coverage factors comes from the knowledge of the normal distribution, which has an exact theoretical function. Predictions can be made as to the probability
that the experimentally derived means (which are what we obtain in a calibration) is within the acceptable confidence interval to the measurand (which is what we want to obtain). The confidence level reflects how far out on the tail would the worst case of the calibrations lie. From the normal distribution, the commonly used confidence levels of 95%, 99%, or 99.7% correspond to intervals using $2\sigma$, $2.58\sigma$, or $3\sigma$, respectively.

The methods and sequence of applying coverage factors and combining uncertainties comprise the main differences in the techniques of estimating measurement uncertainty described below.

**UNCERTAINTY: RSS WORST CASE**

The simplest estimates perform only RSS (taking the square root of the sum of the squares) of worst-case estimates for the effect of each parameters. The RSS combination of uncertainty assumes independence of the parameters, that is, that they occur randomly. Otherwise, if sources of variability are correlated, that is, dependent on each other or yet other parameters, [for example, rising or falling together], the effect of the variation on overall uncertainty may add directly, giving larger error than predicted by the RSS technique.

Also implicit in this technique is the confidence level assigned to the "worst case". The difficulty in interpreting such an estimate is in not knowing the degree of conservatism in the value, specifically, not knowing the coverage factor. The requirements in MIT-STD-45662A [13] state “the collective uncertainty of the measurement standards [assumed here to be the combined uncertainty as defined above] shall not exceed 25 percent of the acceptable tolerance for each characteristic being calibrated”. This would seem to imply that a coverage factor of 4 is required. If this factor is not attained, the calibration complies with the military standard only if the adequacy of the calibration is not degraded, and by documenting deviations from this ratio [13, para. 5.2]. Having to document such deviation would be an unfortunate result if the reason were based solely on uncertainty of the coverage factor.

[Editorial comment: Coverage factor of 4 effectively specifies greater than 99.99% confidence level, which, while desirable for optimum accuracy, is costly and requires much higher quality standards than could otherwise be used. Often it is simply not within the “state-of-the art” to obtain this ratio and still specify an accuracy tolerance that is reasonable for the measurement application. Most commercial suppliers of instrumentation use a 95% confidence level (k factor of 2) in specifying accuracy.]

**UNCERTAINTY ESTIMATION: RANDOM AND SYSTEMATIC COMPONENTS**

Analysis of uncertainty in the literature often separates uncertainty sources into two classes, random and systematic components, which are then given different treatment. The experimentally-derived standard deviations are combined by RSS of the random components, each weighted by their respective contribution to the measurement (see the next section for a description of the weighting). Then they are multiplied by the Student's t distribution, which assumes they all have normal distribution, and corrects the deviation by a factor appropriate to the finite number of readings taken and the desired confidence level.

Separately, the systematic components are corrected as modeled [amplitude linearity factors, temperature dependencies, etc.], then the limits of the remaining dispersions are combined by RSS on the remaining quantities, again, weighted per their effects on the measurement. The result is multiplied by a factor $k/\sqrt{3}$. The $k$ is the coverage factor, corresponding to the same confidence level of the Student’s t correction of the random components. The factor of $1/\sqrt{3}$ is used to convert the RSS of limits to a standard deviation. This is based on the assumption that the systematic variations have rectangular distributions, in other words, that their values are equally likely throughout the spans given by the limits. (The mathematical standard deviation of a rectangular distribution of width $2a$ is $a/\sqrt{3}$.)
Finally, the two types of uncertainties are combined, equally weighted, by RSS. The result is what was defined above as expanded uncertainty, in that inherent in the calculation is the assumed confidence level. However, without detailed knowledge of the respective contributions of the two types of uncertainties (since distributions are assumed as well), it would not be possible to extract the original standard deviation from this measurement, and then incorporate it into another larger analysis which may, for instance, be using a different confidence level.

**UNCERTAINTY ESTIMATION: TYPE A AND TYPE B COMPONENTS**

This is the technique recommended by the International Committee for Weights and Measures, per Working Group 3 of the Technical Advisory Group on Metrology (ISO/TAG 4/WG 3) of the International Organization for Standardization, whose charter was to create a technique for expressing uncertainty which was universal, internally consistent, and transferable. The result was the “Guide for the Expression of Uncertainty in Measurement” [23].

To summarize, uncertainty components should be classified into two categories: Type A, evaluated by the statistical analysis of a series of observations, and Type B, all others, evaluated by other means than the statistical analysis of a series of observations. In other words, the classification is by the method used to evaluate the uncertainty, not on the nature of the types nor their probability distribution. (These types are not to be confused with the random and systematic classifications traditionally used.) The result of each evaluation is a standard uncertainty for each of the components, (actually the squares of the uncertainties, the standard variances, which are central to the assumptions in the Central Limit Theorem) which are then treated equivalently in the calculation of a combined uncertainty. [The key is translating all the uncertainties to a level, independent of assumed distributions, at which apples can be compared to apples.] Type B analysis is necessarily evaluated by judgment, using all relevant information such as “previous measurement data, experience with and general knowledge of the behavior and properties of relevant materials and instruments, manufacturer specifications, data provided in calibration and other certificates, and uncertainties assigned to reference data taken from handbooks”. Generally the evaluator is expected to generate a reasonable standard deviation for the components, square it to get a variance, at which time it can be combined with the other components. There is to be no preference given to one type over the other.

The procedure is summarized in eight steps.

1) State the mathematical relationship between the measurand and the input quantities on which it depends.
2) Determine the estimated value of each input quantity, either on the statistical basis of Type A or by means of Type B, and include corrections for all known systematic effects that significantly influence the final measurand.
3) Evaluate the standard uncertainty (actually the variance) of each input estimate, including the uncertainty of the applied systematic corrections.
4) Calculate covariances of any input quantities which are correlated. “Past experience and general knowledge are key toward estimating the degree of correlation between the input quantities arising from the effect of common influence” parameters.
5) Calculate the estimate of the measurand from the input quantity estimates.
6) Determine the combined standard uncertainty, equal to the positive square root of the sum of terms, each term being the product of the standard variance of that input and the weighting factor which expresses how the measurand varies with changes in that quantity. [Covariances are added, as well.] Mathematically, the weighting factor for an input is the partial derivative of the expression for the measurand with respect to that input. (For
expressions that are messy, it may be easier to
determine each term with a two step process:
Evaluating the expression with all the estimated
inputs, then evaluating it again with the uncertainty
added to one input, keeping all other inputs constant.
The square of the difference will be approximately
equal to the square of the product of the standard
deviation of the input and its weighting factor.)

7) If required, calculate the expanded uncertainty by
multiplying the combined uncertainty with the
coverage factor, selected on the basis of the desired
level of confidence.

8) Report the result of the measurement together with
its combined uncertainty and describe how they were
obtained. Avoid the use of “±” with the uncertainty,
as that form implies expanded uncertainty and very
high levels of confidence. If in fact expanded
uncertainty is reported, state the coverage factor
used, rather than the assumed confidence level.

This can be a lengthy process, likely not to be applied
to each calibration. Practically, rather than making
estimates depending on the unique sensitivities of each
UUT to the various sources, the system specification
insist include the limits or range for the parameters
affecting the uncertainty, as well as the range of
acceptable transducer sensitivities to the parameters.
For example, both the transverse motion generated
by the shaker and the maximum transverse sensitivity
of transducers to be calibrated must be stated. It is
possible to state multiple uncertainty values for various
limits assigned to the parameters and to the transducer
sensitivities to the parameters.

Another note of reality: Not only does the presence of
uncertainty imply doubt about the measurement, even
the estimated value of the uncertainty itself is in doubt,
to the degree of the completeness of the development
of all known contributing affects on the measurement
and of the knowledge and experience of the evaluators.
Combined standard uncertainty is an estimated standard
deviation.

[An important additional issue is stated succinctly in
Reference 23 paragraph 3.4.6: “Although this Guide
provides a framework for assessing uncertainty, it
cannot substitute for critical thinking, intellectual
honesty, and professional skill. The evaluation of
uncertainty is neither a routine task nor a purely
mathematical one; it depends on detailed knowledge of
the nature of the measurand and of the measurement
method and procedure used. The quality and utility of
the uncertainty quoted for the result of a measurement
therefore ultimately depends on the understanding,
critical analysis and integrity of those who contribute to
the assignment of its value.”]

RECALIBRATION INTERVALS
Recalibrations of sensitivity and frequency response
should be performed at regular intervals. The
selection of the appropriate recalibration interval for
an accelerometer involves a trade-off between the
cost of calibration on the one hand and the cost of
measurement uncertainty in the field tests conducted
with the accelerometer.

Ordinarily, recalibrations need be performed only at
regular intervals, which should not exceed 12 months,
if it is known that the accelerometer has not been
used beyond its rated specifications, and if prior
recalibrations show that the sensitivity values are
stable and remain within specification. Most standard
transducers, for instance, show no discernible changes
over five years or more. If the change in a performance
parameter from one recalibration to the next is
outside defined tolerances, the recalibration interval
should be shortened. If the deviation is significant
this information should be transmitted to all those
affected by this change in value of the accelerometer
is used under severe environments, particularly
in shock testing, it may be prudent to use shorter
recalculating intervals initially until sufficient history
has been established that justifies a longer interval.
It is advisable that accelerometers used in shock
testing, other than pyroshock testing, be recalibrated
at intervals not exceeding 6 months. The recalibration
interval for accelerometers used in pyroshock testing should preferably not exceed 3 months. Where the pyroshock tests are expected to generate shock pulse amplitudes close to the accelerometers’ maximum rating (particularly with frequency components near the \( f_n \), it will be wise to perform pre-test and post-test calibrations for each use. Caution: It is common that the shock levels in violent tests are described in terms of heavily filtered data, and so potentially damaging amplitudes at high frequencies may not be reflected in the predictions.

**SUMMARY**

The calibration process requires a great deal of care and an understanding of the underlying physics. There are many sources of variability that can lead the unwary to calibration values that, if not totally erroneous, have a high level of measurement uncertainty.

The physics of acceleration, accelerometers, and calibration techniques has been discussed, along with sources of uncertainty in calibration and use of accelerometers. Suggestions for techniques, procedures and equipment have been given for the estimation and the minimization of uncertainty.