Basics of Shock and Vibration Theory—II

dynamic instrumentation Σ

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The first article (Sept., 1962, issue) in this series—originally planned for two parts but now extended to three—explored simple harmonic and complex motions and various aspects of their dynamics and damping. This installment takes up . . .

Aperiodic Motion: Random Vibration

Random vibration is a continuous oscillating motion whose instantaneous amplitude can be predicted only on a probability basis. It may be considered as being composed of a continuous spectrum of frequencies whose individual amplitudes are varying in a random manner. Random motion is obviously aperiodic and is described mathematically in terms of statistics, rather than trigonometeric functions. Figure 14 is a plot of random acceleration vs. time. At any given instant (t), the probability that the acceleration value is between a_o and $a_o + da$ is defined as: p (a) da. For a normal process:

$$p(a) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{a^2}{2\sigma^2}\right) \qquad (20)$$

This equation, known as a Gaussian or Normal Distribution, is plotted in Figure 15 as a function of a.

The probability that the instantaneous value of acceleration is between a_1 and a_2 is given by:

$$\int_{a_1}^{a_2} p(a)\,da = \frac{1}{\sigma\sqrt{2\pi}} \int_{a_1}^{a_2} \exp\left(-\frac{a^2}{2\sigma^2}\right) da$$

which is the shaded area under the curve of Figure 15.

Probability Density

Since the amplitude probability is the product of p (a) and an acceleration value, p (a) is commonly referred to as the probability density. Equation 20, then, describes a Gaussian probability density function. (The only vibrations considered here will be those which are described by Equation 20. It will also be assumed that the random vibration is stationary -- that is to say that its statistical properties are unaffected by a translation of the time axis.)

The quantity σ is defined as the root mean square deviation of the instantaneous acceleration from the mean acceleration value. For random acceleration, the mean acceleration value is zero, so that σ reduces simply to the rms value of the instantaneous acceleration.

The probability density curve is usually normalized -- that is, the scales are so adjusted that the total area under the curve is unity, which is to say that the total area under the curve represents certainty, with a probability of 1.

Referring to Figure 16, the probability that the instantaneous value of acceleration is between $\pm a_1$ is equal to the shaded area under the normalized probability density curve.

Random Amplitude Sine Wave

From a damage standpoint, peak values of acceleration may become more important than instantaneous values. Suppose the random vibration signal is passed through a narrow bandwidth filter. The result will be a

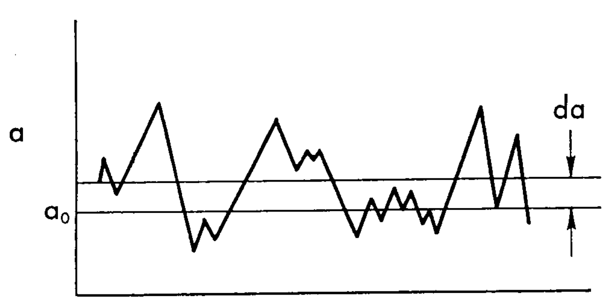

Figure 14

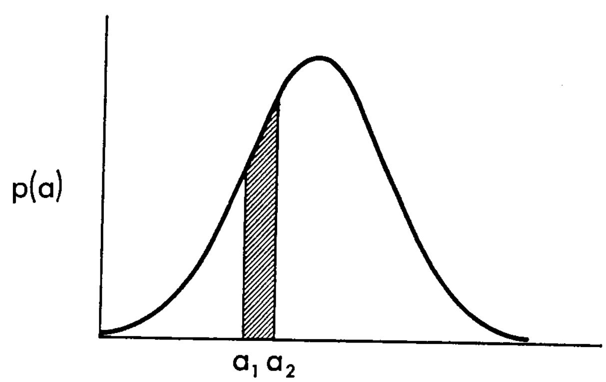

Figure 15

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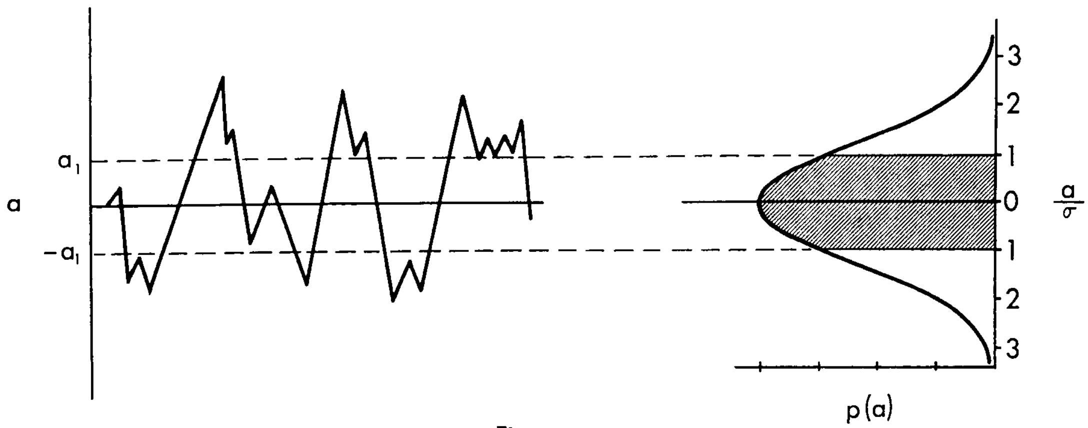

Figure 16

single frequency wave with randomly varying amplitude, or random amplitude sine wave.

For such a wave, the probability of an acceleration peak having a value between a_o and $a_o + da_p$ is defined as:

$$p(a_p)\,da_p$$

Where $$p(a_p) = \frac{a_p}{\sigma^2} \exp\left(-\frac{a_p^2}{2\sigma^2}\right) \qquad (21)$$

This equation, known as a Rayleigh distribution, is plotted in Figure 17 as a function of a_p.

From Equation 21, the probability that the peak value of acceleration is between a_1 and a_2 becomes

$$\int_{a_1}^{a_2} p(a_p)\,da_p = \frac{1}{\sigma^2} \int_{a_1}^{a_2} a_p \exp\left(-\frac{a_p^2}{2\sigma^2}\right) da_p$$

which is the shaded area under the curve.

By analogy with the previous discussion of instantaneous acceleration, $p(a_p)$ is a probability density function for peak accelerations, or equivalently, of the envelope of the random sine wave.

Referring to Figure 18, the probability that the peak acceleration is between o and a_1, is equal to the shaded area under the normalized probability density curve.

A single frequency component of a random vibration will vary randomly in amplitude. This component cannot, therefore, be specified by its peak value. Instead, its root mean square (rms) value, which does not vary with time, must be used.

The rms value of a single frequency wave is easily defined. Random vibration, however, contains a continuous distribution of frequencies. Since any actual bandpass filter used in practice has a finite bandwidth, it will pass frequencies adjoining the center frequency of interest. To determine the contribution of a single frequency, it is necessary to divide the filter output by its bandwidth. If output is in rms gs, it is divided by the square root of the bandwidth (yielding $\frac{g}{\sqrt{cps}}$). If output is in mean squared gs, it is divided by the bandwidth in cps (yielding $\frac{g^2}{cps}$).

Power Spectral Density Plot

Random vibration is normally plotted in the latter units as a function of frequency. Figure 19 illustrates a typical power spectral density plot. (Power, the rate of doing work, is proportional to the square of the vibration amplitude. Hence a plot $\frac{g^2}{cps}$ vs. frequency shows the power distribution of the vibration as a function of frequency.)

The shaded area under the PSD curve is given by

$$\bar{a}^2 = \int_{f_1}^{f_2} G(f)\,df \qquad (22)$$

and represents the mean squared acceleration between f_1 and f_2.

The rms acceleration (between frequencies f_1 and f_2) is therefore equal to the square root of the shaded area of Figure 19.

Random vibration which exhibits a constant accel-

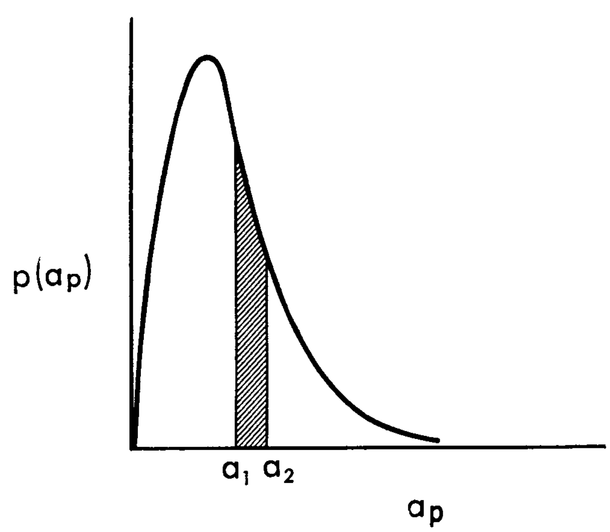

Figure 17

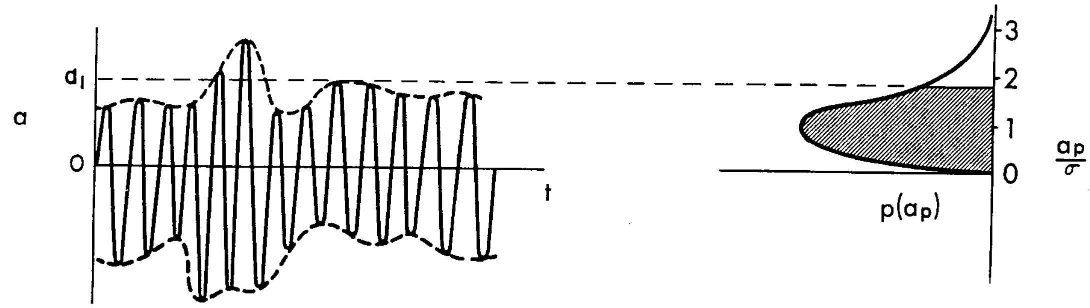

Figure 18

eration density is called a white noise. In the case of white noise, Equation 22 simplifies to:

$$\bar{a}^2 = G_o B$$

and $$a_{rms} = \sqrt{G_o B} \qquad (23)$$

when G_o = constant acceleration density and B = bandwidth under consideration.

Random vibration is important because it it frequently encountered in nature. Rocket engines are typical random vibration generators. Fortunately, the rms acceleration level, which is easily measured, has statistical significance. The amplitude of random vibration is most often specified as an rms acceleration over a given bandwidth and as an acceleration density vs frequency.

Dynamics

The behavior of the system illustrated in Figure 8 (EQ, Sept., 1962, Pg 22) was examined for a forced vibration of the mass. If the forcing function is applied instead to the support, the resultant motion of the mass or transmissibility of the system becomes:

$$T = \sqrt{\frac{1 + \left(2h\frac{\omega}{\omega_n}\right)^2}{\left(1 - \left[\frac{\omega}{\omega_n}\right]^2\right)^2 + \left(2h\frac{\omega}{\omega_n}\right)^2}} \qquad (24)$$

If the forcing function is random sine wave (single frequency) the resultant motion of the system is the same as for a pure sinusoidal input. The product of the input excitation and the system transmissibility is $a = a_o T$, where a = resultant system acceleration, a_o = input acceleration (of support), T = transmissibility.

In terms of squared acceleration:

$$\bar{a}^2 = \bar{a}_o^2 \; T^2$$

where a^2 = mean squared resultant acceleration and $\bar{a}^2$ = mean squared input acceleration.

Since only a single frequency is present, $\bar{a}^2$ is equal to G(f), the mean square acceleration density. If, instead of a single frequency, the input is a random vibration the overall mean squared response is the sum of the mean squared responses to the component frequencies:

$$\bar{a}^2 = \int_o^\infty T^2 \; G(f) \, df \qquad (25)$$

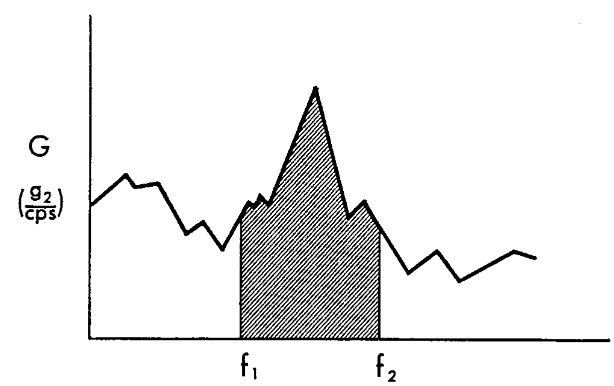

Figure 19

Environmental Quarterly

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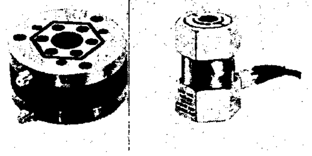
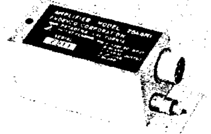


FORCE GAGES

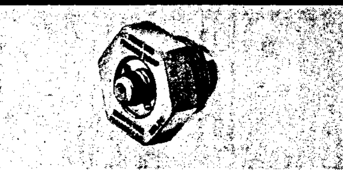
MODEL 2106 FORCE GAGE incorporates three force tranducers in the same plane to accurately simulate point loading. This minimizes concern about shear loading and permits the convenience of the center mounting hole. Maximum dynamic load is 10,000 lbs. with sensitivity of 7 pk mv/pk lb. Gage stiffness is 2 x 10^7 lb. Works only in dynamic compression.

MODEL 2103-2104 FORCE GAGES work in both tension and compression from 5 grams to 5,000 lbs. at frequencies from 2 cps. to 6,000 cps. Sensitivity is from 375 to 30 pk mv/pkg and stiffness is from 5 to 14.2 x 10^6. This high stiffness permits high frequency applications and use as load carrying members.



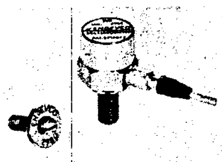
NEW CHARGE AMPLIFIERS

Model 2640 Amplifiers are a new series of miniature, all transistor charge amplifiers designed for operation in severe flight environments. All welded module construction coupled with highest quality control standards assure dependable performance under the most demanding conditions. . . . For example, the Model 2646MI features instantaneous recovery from as much as ten times full scale overload, system sensitivity virtually unaffected by length of input cable, flat (±5%) frequency response from 3 cps to 10 Kcps, operation from unregulated 28 V supply and broadband residual noise better than 40 db down from full scale output. Unit is 1" x 1.115" x 2.165" and weighs 95 grams.



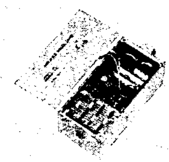
DYNAMIC PRESSURE PICKUP

MODEL 2500 SERIES PRESSURE PICKUPS are piezoelectric self-generating transducers which feature large outputs up to 50 pk-mv/pk-psi, very fast rise time of approx. 5 microseconds (0 to 90%) and flat frequency response of 2 to 10,000 cps (±5%) with 1000 megohm load. Static pressure components do not produce any voltage output. Variations in liquid or gas pressure produce voltage outputs in direct relation to the input variations. Model 2501 series for general applications — Model 2503 for turbulence measurements and very small size. These features make them suited for acoustic and blast measurement.



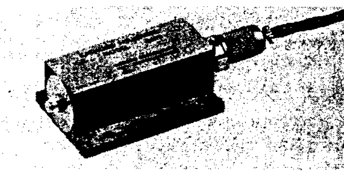
INSULATED MOUNTING STUDS

MODELS 2980B* (10-32/10-32) and 2983B* (10-32/¼-28) MOUNTING STUDS may be used with all old and most new models of Endevco Accelerometers. These studs provide complete electrical isolation from structural ground. Removable studs eliminate the complete rebuilding required when an integral stud is damaged.
*(Reg. U.S. Pat. Office)



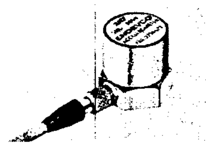
LABORATORY A.C. INPUT AMPLIFIER

MODEL 2614B AMPLIFIER has the greatest versatility of all Endevco amplifiers in shock and vibration work. It provides a very high input resistance of 1,000 megohms and gains of 1, 3 or 10 into an output load of 2500 ohms or more. Frequency response is flat to below 2 cps. A three decade switch is provided on the input to allow transducer standardization by shunt capacity across the input. These same features are available in a 3-channel rack mounted version — the Model 2616B.



CATHODE FOLLOWERS

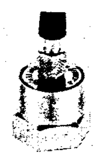
MODEL 2619 CATHODE FOLLOWER operates in temperature range of −65°F to +500°F, an ideal combination for the Endevco Model 2242C high temperature Accelerometer. The Model 2608 operates in temperature range of −20°F to +170°F. Both cathode followers provide input impedance of 100 megohms with flat frequency response from 2 cps to 20 Kc. They are shock resistant. Dimensions exclusive of mounting bracket and cable are 2.5" x 1" x 1" dia. for the Model 2619 and 2-1/64" x 1" dia. for the Model 2608.



HIGH OUTPUT — SUBMARINE LOW NOISE TESTS

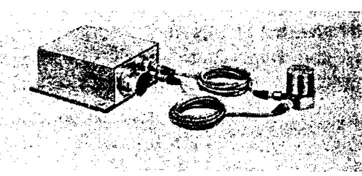
MODEL 2217 ACCELEROMETER provides a high sensitivity of 72 pk-mv/pk-g and a wide frequency range (resonance frequency is 35 Kc). A hermetic seal extends its usefulness and the weight is only 1.1 ounce. An even higher sensitivity of 350 pk-mv/pk-g is available in the MODEL 2219 ACCELEROMETER.

Endevco Model 2870 Measuring System for very low noise tests incorporates the (Model 2217 Accelerometer) with shielding, shielded preamplifier, and battery power supply.



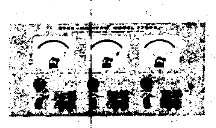
SHOCK ACCELEROMETER

MODEL 2225 ACCELEROMETER accurately measures shock and impact transients because of high resonance frequency of 80 Kc (Nominal). Dynamic range of this new transducer is a maximum sinusoidal 10,000 g and a maximum shock of 20,000 g with a 75 μsec half sine pulse. This high g range together with the low sensitivity make it ideal for high level shock testing. Nominal sensitivity is 0.6 pk-mv/pk-g with 300 pf external capacity. This new Endevco product weighs only 0.46 oz. (13 grams) and is 0.52" high with a 9/16" hex base.



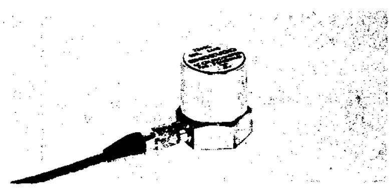
TELEMETRY SIGNAL CONDITIONERS

MODEL 2630 SERIES are all transistorized, one-package systems with variable features to meet the varied requirements of telemetry systems. Filtering, biasing, limiting, extra power regulation, extra amplification, and calibration insertion are all available. A unique feature is that the filter can be in front of the amplifier. This is possible because of the large amplifier which is used (see Model 2620 Amplifier). For optimum accuracy and reliability consider a 2630 Series unit with a Model 2242M5 Accelerometer and a Model 2980B Insulated Stud.



COMPLETE LABORATORY SYSTEM

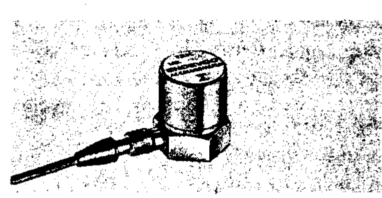
MODEL 2702C DYNA-MONITOR® is an integrated system which saves time and trouble in laboratory work. **Driven Shield** cable allows the 2702C to be remotely located from a shaker without capacitive signal attenuation. **Power Amplifier** will drive any high frequency galvanometer directly. **Precision VTVM** readout can also be calibrated in g's by simple procedure. Large meter for each of three channels is easy to read and a relay for "shut down" is optional. **Integrated Power Supply** eliminates lash up and saves space. Provision is made at front for **convenient oscilloscope output** and filter insertion. **High Gain** is available with 10 mv input giving full scale reading and 6.0 volts rms output & 85 Ma peak.



HIGH TEMPERATURE ACCELEROMETER

MODEL 2242C ACCELEROMETER measures vibration and shock at +500°F (and even higher for brief exposures) without cooling or correction. It is very stable and has no sensitivity change (hysteresis) after successive heat runs. Sensitivity changes ±5% or less from −320°F to +500°F. These unique characteristics are made possible by Endevco's PIEZITE® Element Type II. Sensitivity is 10 pk-mv/pk-g and resonance frequency is 33 Kc.

MODEL 2242-M5A ACCELEROMETER incorporates provision for inserting a calibration current through a precision resistor built into the base of the accelerometer. The case is hermetically sealed by resistance welding. A double-walled case provides protection from the effects of transient thermal and pressure pulses.



SINGLE ENDED COMPRESSION ACCELEROMETERS

MODEL 2213C FOR USE TO +350°F
MODEL 2233 FOR USE TO +350°F
MODEL 2234 FOR USE TO +500°F

These new accelerometers have a unique single ended compression design which completely separates the active elements from the case. High sensitivity at high temperature is another outstanding characteristic.

Sensitivity is 34 pk-mv/pk-g with high capacity of 1200 pf including 10 ft. cable.

Mounted resonance frequency is 35 Kcps. The Models 2233 and 2234 feature 3% maximum cross axis sensitivity and hermetic sealing. Calibration at 500°F is provided for each Model 2234.

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