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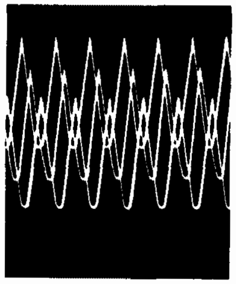

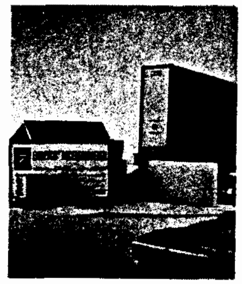

A New Method of Evaluating the Acoustic Response of Piezoelectric Accelerometers

by
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INTRODUCTION

Piezoelectric accelerometers are designed to respond only to accelerations applied to their mounting surfaces. Frequently they are used to measure vibrations where another severe environmental factor, acoustic noise, is present. In the case of the Saturn space booster, for example, acoustic pressures may exceed 170 db, and the question has arisen whether these pressures can cause a significant error in the acceleration measurements.

A great deal of testing has been done to provide an experimental answer to this question. A measurement of the ratio of accelerometer output to acoustical input, when no acceleration is applied to the accelerometer base, would permit answering it for any particular case. Unfortunately, such tests are subject to a multitude of problems, some of them common to nearly all acoustic tests, and some peculiar to the testing of accelerometers. Various tests have yielded conflicting results, most of them indicating a ratio too high to be acceptable in intense acoustic fields.

In this paper, a theory is developed for predicting the approximate acoustic response of piezoelectric accelerometers. This theory is used to show the limitations of existing tests, and to show that the acoustic response of any reasonably well designed piezoelectric accelerometer may be safely neglected even in "Saturn type" testing.

EARLY TESTING AND SOME TYPICAL RESULTS

Most of the problems which accelerometer testing shares with acoustic testing in general have been well understood for some time. It is well known, for example, that the intensity of the pressure field applied to a test object is not easily determined, since the

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presence of the object itself changes the field in its vicinity when its size approaches the wavelength of the sound. Calculated values for the magnitude of this effect, called "diffraction," are available in the literature.[1] For an unmounted accelerometer, the effect is maximum for wavelengths of about one inch, or approximately 10 kHz, and may cause an increase in the local sound pressure level of as much as 8 db, depending on the exact nature of the pre-existing field.

Some of the problems peculiar to accelerometer testing are not so well understood. That is particularly true of the question of how to assure that "no acceleration is applied to the accelerometer base." Historically the earliest and still the simplest testing approach is to hang the accelerometer in an acoustic test chamber by means of a very soft suspension so that it is not subjected to the vibrations of the chamber walls.[2] Usually the chamber is energized with some form of random noise, both to approximate the noise in which the accelerometer will eventually have to operate, and to average out the rather lumpy frequency response of most high intensity acoustic chambers. The results of this test are quite repeatable, and do not depend to a significant degree on the nature of the test chamber, whether it be a plane wave tube, or a reverberation chamber. Most of the available data on accelerometer acoustic response are based on this test. Typical data for a single ended compression piezoelectric accelerometer are given in Table 1, with responses shown in equivalent g.

Because of equipment limitations, this particular test was run in a plane wave tube adjusted for equal energy per octave, rather than white, excitation. The shape of the spectrum has no effect on frequency response measurements, which are made with the same filters that are used to adjust the chamber equalization, but it does affect the gross output of a unit whose frequency response is not flat. The "white" response at the bottom of Table 1 was calculated by making an appropriately weighted rms summation of the octave band data, so that they are easily compared to the theoretical results that follow.

Table 1 - Response of a Rubber Band Supported Piezoelectric Accelerometer in a Plane Wave Tube

Octave Band, Hz	Response (g/psi)
75-150	0.38
150-300	0.38
300-600	0.77
600-1200	1.54
1200-2400	3.07
2400-4800	6.15
4800-10,000	12.50

Measured overall response at 150 db, equal energy per octave: 0.52 rms g.
Calculated overall response at 150 db white spectrum: 0.88 rms g.

These data would cause no concern to anyone at the 130 db sound pressure levels common around jet aircraft. At 150 db, they are still more or less acceptable. But for 170 db (or higher) encountered in "Saturn type" testing, they indicate an output equivalent to 5 g or more from acoustic response alone It then becomes worthwhile to spend some time to analyze the whole question of accelerometer acoustic response in more detail, and to find if these commonly accepted data are not unduly pessimistic due to errors inherent in the test.

THEORETICAL ANALYSIS OF ACCELEROMETER RESPONSE TO AN ACOUSTIC FIELD

Of the many effects which contribute to the acoustic response of an accelerometer, some, such as diffraction, depend on exceedingly complex interactions (between the test object and the chamber for example) and can be estimated only to the extent of the maximum effects to be expected. Many others, however, can be calculated very closely, and provide considerable insight into the mechanisms involved. For illustration, an analysis will be made of a typical Single-Ended Compression accelerometer, the ENDEVCO® Model 2213C (shown schematically in Figure 1). To simplify calculation, responses will be calculated for a "white" spectrum, 75 Hz to 10 kHz, 0.1 psi rms pressure level (approximately 150 db SPL) and all responses will be assumed in phase, corresponding to the worst case.

The sources of acoustic response will be covered point by point, the calculable pressure response first, and then the variable effects.

Basic Pressure Sensitivity

Acoustic Pressure on Crystal:

To a first approximation, the pressure sensitivity of the crystal is frequency independent. The pressure response of the crystals is determined by the piezoelectric constants. For the crystals used in the Model 2213C accelerometer the pressure sensitivity is about 1.5 g/psi. However, only a small portion of the ambient pressure is transmitted to the crystal. A conservative calculation of the deflection of the accelerometer cover shows that its stiffness is sufficient to provide an attenuation of at least 55 db. Anticipated output from this source is thus less than .003 g/psi.

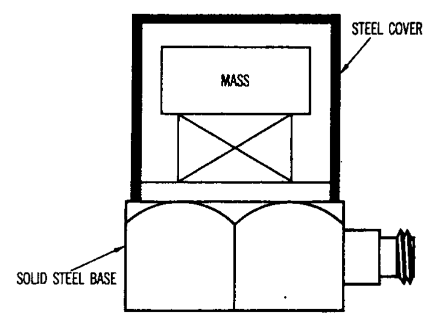


FIGURE 1
SCHEMATIC CUT-AWAY OF TYPICAL S.E.C. ACCELEROMETER, eg. MODEL 2213C

Crystal Output Due to Radial Strain Applied Through the Accelerometer Base:

Stresses applied to the base of a compression accelerometer are transmitted to the crystal and produce outputs through the transverse charge coefficient, d_{31}. The stress distribution in the base of a mounted accelerometer is too complicated for easy computation of this effect, but it is easily measured by applying quasi-static pressures to the outside of the accelerometer. Tests conducted at the Space Technology Laboratories in El Segundo[3], using air as the pressure medium, and in the engineering laboratory at Endevco, using oil, show a low frequency pressure sensitivity of .03 g/psi for the 2213C, linear with pressure level. It appears that the "strain gage" effect is responsible for practically all of this output, since no other low frequency effects of significant size can be seen. (In the earlier designed "Isolated Compression" accelerometers, forces transmitted through the preload spring boost the response to about 2 g/psi, but no such force acts in the SEC design.)

Acceleration of Sensing Elements Due to Vertical Base Strains:

The elastic deformation of the base causes motion at the bottom of the crystal and an output is produced due to inertial loading of the mass. This is the only frequency sensitive component of the accelerometer's basic pressure sensitivity. When the accelerometer is subjected to hydrostatic pressure, the height of the base is decreased by approximately

$$\Delta h \ + hp \ \left(\frac{1 - 2v}{E}\right)$$

where h = the base height, p = the pressure, E = the modulus of elasticity, and v = Poisson's ratio. When such height changes occur sinusoidally, an acceleration of the top surface equal to $4\pi^2 f^2 \ \Delta h \sin 2\pi ft$ is produced. This acceleration is negligible compared to .03 g/psi at low frequencies, but at 10 kHz it amounts to .04 g/psi for a 2213C, slightly more than the direct pressure sensitivity; and it increases rapidly, rising as the square of the driving frequency.

Variable and Second Order Effects

Wave Length Effects Other Than Diffraction:

The two effects just described involving base volume strains may produce an additional frequency sensitive component if the pressures applied to the top of the base (actually to the top of the cover, and transmitted down the case walls) and those applied to the sides of the base get out of phase, so that stress effects become additive rather than compensating. It's possible for that to happen near 10 kHz, but not in any predictable fashion. This term is best treated as an estimated maximum effect, similar to the diffraction effect. In the case of the "strain gage" term, the sensitivity may increase by a maximum ratio of $(1 + v)/(1 - v)$ or about a factor of 2; in the case of the acceleration term, a maximum ratio of $(1 + 2v)/(1 - 2v)$ or about a factor of 4. The sum of the terms will increase a maximum of about 10 db, or perhaps 5 db broadband. These effects, like the diffraction effects, would be neglibible below about 2 kHz, and would be strongly dependent on transducer orientation and chamber interactions.

Mechanical Resonances:

Accelerometer resonances are not a significant source of acoustical output, at least in the 2213C and probably not in other ENDEVCO® SEC accelerometers. Tests using mechanical and acoustical exciatation on this accelerometer have thus far succeeded in identifying a low Q case resonance at about 22 kHz, but no other axial, transverse, or radial mode resonances below 30 kHz. None of the identified resonances could have more than a 1 db effect on measurements taken below 10 kHz.

Cable Noise:

Cable noise test results indicate that this effect may be neglected for accelerometers with more than about 10% of the 2213C's charge sensitivity of approximately 50 pC/g.

Summation of Accelerometer Outputs

The discussion above reveals that the 2213C has two significant sources of pressure sensitivity: a radial strain effect which is fundamentally independent of frequency, and a vertical strain effect which is strongly frequency dependent. By properly summing these over a frequency range, and then applying a reasonable factor for the variable effects of stress phasing and diffraction, it is possible to compute an approximate value for the accelerometer acoustic response. For a discrete frequency test the pressure response would be:

$$a_g\ (\text{equiv.}) = (.03 + 4 \times 10^{-10}\ f^2)\ \text{g/psi}.$$

(This response is shown in Figure 2.) The response, in rms g, to broadband "white" excitation is computed as follows:

$$a_g\ (\text{rms}) = \sqrt{\frac{1}{\Delta f}\int_{f_1}^{f_2} (a_g\ \text{equiv.})^2\ df}$$

$$= \sqrt{\frac{1}{\Delta f}\int_{f_1}^{f_2} p^2\ (.03 + 4\text{x}10^{-10} f^2)^2\ df}$$

for $f_1 = 75$ Hz, $f_2 = 10$ kHz,

$$a_g\ (\text{rms}) = (4.6 \times 10^{-2})\ p$$

The spectrum level constant p, is evaluated from:

$$p_{rms} = \sqrt{\frac{1}{\Delta f}\int_{f_1}^{f_2} p^2\ df} = p$$

for 150 db, 75 Hz to 10 kHz, white excitation, a (rms) = .0046 rms g, basic pressure response. Allowing a maximum of 1 db for resonance effects and 10 db for diffraction and phasing effects brings the maximum expected response to about .02 rms g.

The data in Table 1 were taken from a freely supported test of a 2213C in an acoustic field. Corrected to a white spectrum, they show a response of 0.88 rms g, which is so large that .02 rms g could be neglected compared to it. If the theory just presented is valid, then there must be a large error signal in a freely supported test which is not a legitimate part of the acoustic response of the accelerometer.

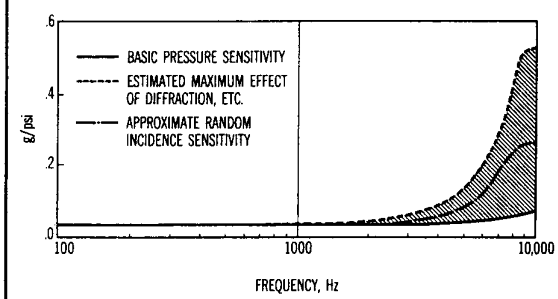
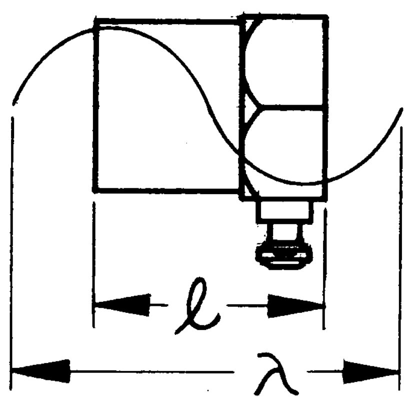


FIGURE 2

THEORETICAL ACOUSTIC RESPONSE OF MODEL 2213C, RIGIDLY MOUNTED

Error Signals in the Freely Supported Test

The only source of output which might qualify to explain the 0.88 rms g test result, without being a legitimate source of acoustic response is rigid body motion of the accelerometer. This possibility can be checked by another computation:



If the accelerometer has a mass m and cross sectional area A, and is located in a traveling wave tube whose pressure at some point is characterized by $p = p_0 \sin 2\pi ft$, then the accelerometer will experience an acceleration due to the pressure gradient across it. Neglecting diffraction effects, it will be given by:

$$a = \frac{F}{m} = \frac{Ap_0}{m} \left[\sin 2\pi ft - \sin (2\pi ft + \phi) \right]$$

from the figure:

$$\phi = \frac{2\pi \ell}{\lambda} = \frac{2\pi \ell}{c/f} = \frac{2\pi \ell f}{c} \qquad (c = \text{sonic velocity})$$

Substituting for ϕ and expanding:

$$a = \frac{Ap_0}{m} \left[(1 - \cos \frac{2\pi f\ell}{c}) \sin 2\pi ft - \sin \frac{2\pi f\ell}{c} \cos 2\pi ft \right]$$

The rms amplitude of this acceleration is given by:

$$|a| = \frac{1}{\sqrt{2}} \frac{Ap_0}{m} \sqrt{(1 - \cos \frac{2\pi f\ell}{c})^2 + \sin^2 \frac{2\pi f\ell}{c}} = \frac{Ap_0}{m} \sqrt{1 - \cos \frac{2\pi f\ell}{c}}$$

For broadband excitation:

$$a\ (\mathrm{rms}) = \sqrt{\frac{1}{\Delta f}\int_{f_1}^{f_2} |a|^2\, df}$$

when $f_2 >> f_1$,

$$a\ (\mathrm{rms}) = \frac{Ap_0}{m}\sqrt{1 - \frac{c}{2\pi \ell f_2}\sin\frac{2\pi \ell f_2}{c}}$$

In this case, $p_0 = p\ (\mathrm{rms})\ \sqrt{2}$

For a 2213C, $A = 0.30\ \mathrm{in}^2$, $mg = 0.6$ pound, $\ell = 0.8$ inch.

Taking $p\ (\mathrm{rms}) = 0.1$ psi, $f_2 = 10$ kHz and $c = 13{,}200$ in/s gives

$a_g\ (\mathrm{rms}) = 0.76$ rms g.

This calculated value for motional output, diffraction effects neglected, is just 1 db less than the experimental value quoted earlier in Table 1. The degree of correlations is even more impressive when the frequency distributions are compared, Figure 3. The discrepancy is almost entirely in the top octave, and is probably due to diffraction effects.

As mentioned earlier, these results apply to a plane wave test. The situation in a reverberant chamber is more complicated and more variable, but the similarity of results mentioned in the description of early testing indicates that the difference is of little practical importance in most accelerometer testing.

TESTING WITH MOTION COMPENSATION

Two sets of calculations have been made so far. One shows that the accelerometer acoustic output should be quite small compared to experimental results using the freely suspended approach. Another shows theoretically that rigid body motion alone can account for the observed output in such a test. These support each other rather well, but to actually prove the case requires conclusive empirical data from a more satisfactory test. This improved test will have to make provisions for eliminating the rigid body motion of the test accelerometer, mechanically, electronically, or both.

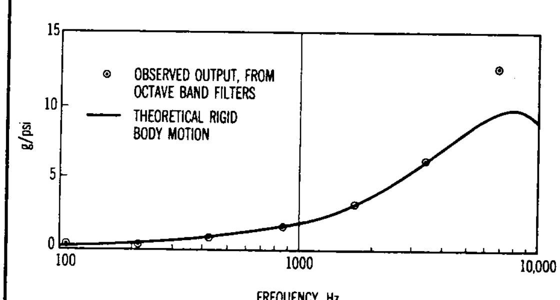


FIGURE 3

ACOUSTIC RESPONSE OF 2213C, RIGIDLY MOUNTED
(NOTE CHANGE OF SCALE FROM FIG. 2)

Recent Testing at Sandia Laboratory

The Sandia Laboratory, Albuquerque, New Mexico has recently reported the results of a series of accelerometer acoustic tests which used mechanical and electronic means simultaneously to eliminate the effects of motion at the accelerometer mounting surface.[4] The principal means of motion suppression was mounting the accelerometer to a 150 lb. steel mass by means of an aluminum adapter (Figure 4). In use, the spud at the apex of the conical adapter extends through a small clearance hole in the wall of a reverberent chamber so that the accelerometer is inside, and the mass, supported by bungee cords, is outside. This arrangement avoids direct acoustic excitation of the large inertial mass as well as the diffraction effects which would be fully developed around a mass of that size at about 1 kHz if it were inside the chamber.

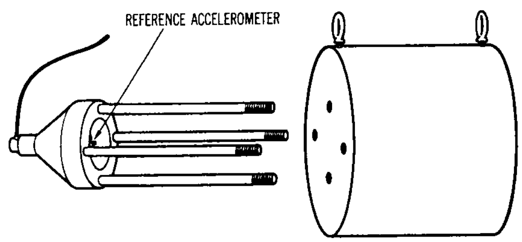


FIGURE 4
SANDIA ACCELEROMETER MOUNT FOR ACOUSTIC MEASUREMENTS (After R. J. Klingler)

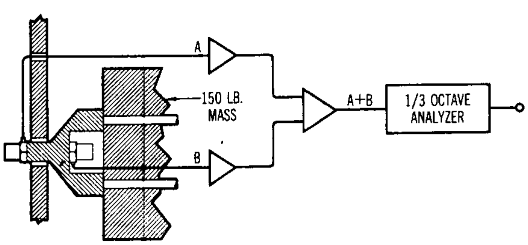


FIGURE 5
BLOCK DIAGRAM OF SANDIA ACCELEROMETER SIGNAL CONDITIONING EQUIPMENT

With only about 1/2 square inch exposed to the pressure inside the chamber, an infinitely rigid mass of 150 pounds would be excited to only about .0004 rms g at 150 db SPL, a vibration small enough to neglect. Unfortunately, steel is not infinitely rigid, and in fact a mass of the size used for these tests would be expected to have resonances near 10 kHz. As an added precaution, therefore, the aluminum adapter was provided with an internal mount for a monitoring accelerometer, outside the acoustic chamber. The outputs of the two accelerometers were then added electronically so that their acceleration outputs would cancel one another, and the net output would be only the acoustic output of the test accelerometer (Figure 5).

The results to be expected when an accelerometer (say, a 2213C, to continue the illustration) is tested with the equipment just described can be computed in much the same way as the acoustic sensitivity of the accelerometer alone and the results of such a computation are of some interest. The Sandia results are the best acoustic data known to the author so whatever short comings they show are a good indication of the general difficulty of the test problem.

All the steps in the calculation are the same as before, except that the estimation of the accelerations caused by vertical strains must include the strains in the aluminum fixture between the test accelerometer and the reference accelerometer. The fixture is loaded on the end by the acoustic pressures in the reverberant chamber, and experiences a change in length given by:

$$\Delta \ell = k \ell p \frac{1}{E_a}$$

where ℓ = the distance between the two accelerometers, p = the pressure, E_a = the Young's modulus of aluminum, and k = a dimensionless strain constant determined by

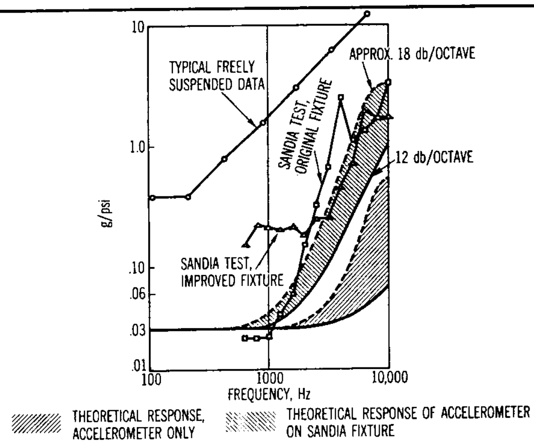


FIGURE 6

COMPARISON OF THEORETICAL RESPONSE WITH VARIOUS TEST DATA, ILLUSTRATING THE FIXTURE PROBLEM the geometery of the fixture. For a uniform cylinder k = 1; for the fixture used k is given by an approximate application of the theory of elasticity as about 0.55. Because of the requirement for placing the reference accelerometer in a well shielded position outside the chamber ℓ was 1.75" for the fixture used, several times the height of the accelerometer base. The computed respons of the total combination is:

$$A_g \text{ (equiv)} = (.03 + 9.9 \times 10^{-9} f^2) \text{ g/psi}$$

Corrections for variable high frequency effects are limited to about 10 db, mostly due to diffraction, since the stress phasing effects are largely limited to the accelerometer itself.

Experimental data from two Sandia runs on the same accelerometer, 2213C S/N FA80, are shown replotted together in Figure 6. Shown also in this figure, for comparison, are the expected response just computed, the expected response of the accelerometer alone, and the measured response of the freely suspended accelerometer. The peaks in the experimental data, near 4 kHz on one curve and near 6.3 kHz on the other, were identified[4] as transverse resonances of the test fixture causing an unbalanced output through the cross axis sensitivities of the two accelerometers. The amplitude was reduced and the frequency increased between the two tests because of a modification to the fixture. The change in apparent low frequency response was caused by a simultaneous modification to the electronics which gave an improvement in the high frequency characteristics of the nulling circuit, but at some sacrifice in the noise level. Electrical noise is a perpetual problem in this kind of test, since signals are in the microvolt region.

The agreement between theory and experiment in this case is quite satisfactory, especially considering that no attempt has been made, for lack of adequate data, to include resonance effects in the theory. In the case of the accelerometer alone, such a correction, though minor, has been made, so the agreement should be better.

Other Experimental Data

Two other recent experiments provide general support for the theory just presented. In January 1963, the Endevco Corporation independently conducted an acoustic test using an acoustically shielded reference accelerometer in the chamber with the test accelerometer (to achieve close mechanical coupling). Cancellation of the acceleration signals was accomplished through a passive network, to avoid phase shift errors, and cross axis effects were minimized by choosing accelerometers with very low cross axis sensitivity, and by conducting the test in a plane wave tube so that cross axis excitation would be low. When the tube was excited at 150 db SPL, equal energy per octave 75 Hz - 10 kHz, the output of a 2213C was .01 rms g: about 2% of the output of an uncompensated freely suspended accelerometer, and roughtly comparable to the theoretical response. An exact comparison was not possible because the signal level out of the amplifier was too low for the octave band analysis equipment which was available at the test facility.

A much different approach to the problem was taken in a test conducted at the David Taylor Model Basin.[5] Pressure sensitivity measurements were made over an extended frequency range using water as a pressure medium. The high sonic velocity in water multiplies wavelengths by a sufficiently large factor, compared to air, so that freely suspended testing of accelerometers may be conducted with little or no concern for pressure gradients or diffraction. When tests were conducted by this means up to 7 kHz on a compression accelerometer similar in design to the 2213C, curves were obtained which were similar in shape and magnitude to those computed for the 2213C using the basic pressure part of the theory. The utility of this clever test for acoustic evaluations is unfortunately limited by its avoidance of the legitimate effects (diffraction, stress phasing, etc.) which occur at short wavelengths.

All of these data support the conclusion that no important source of acoustic response has either overlooked or underestimated in the theoretical analysis.

ESTIMATION OF ACOUSTIC RESPONSE OF ACCELEROMETERS OTHER THAN MODEL 2213C

Because stress distributions in accelerometers will be quite complicated, it will not in general be practical to compute stresses and strains rigorously, but it should not be necessary. Acoustic outputs of the order indicated for the 2213C even with the pessimistic assumptions made as to phasing, can be neglected compared to the structural motions which will accompany an acoustic field. At frequencies approaching 10 kHz it is difficult, in fact, to design a structure which will not have motions large compared to the acoustic output. At lower frequencies where blast effects, for example, operate, that may not be true. But at low frequencies it is comparatively easy to design adequate test gear.

The approximate basic pressure sensitivity of an accelerometer can be taken as the sum of its experimentally determined low frequency sensitivity, plus a reasonably pessimistic approximation to the motion experienced by the sensing element through elastic deformations. Some judgement will be required in individual cases, but the results are not critical. Any motional effects should be multiplied by the mechanically determined resonance curve, if it is significant in the frequency range of interest.

The maximum probable response is then determined by multiplying the basic pressure sensitivity curve by factors for stress phasing (if present–some judgement again required) and diffraction. Both of these effects will peak in the vicinity of wavelengths equal to one or two times the transducer dimensions (the former nearer two, the latter nearer one). When these wavelengths are near or above the maximum frequency of interest, a satisfactory solution is to sketch in curves similar to those shown in this report. For larger transducers (or higher frequencies) an acoustical textbook should be consulted.

If the accelerometer has a charge sensitivity less than about 5 pC/g, it may be necessary to make an allowance for cable noise. This factor is very difficult to assess, since cables are typically many wavelengths long even at frequencies below 1 kHz. The best that can be expected in most cases is an estimate of the maximum probable effect. Usually, that will be enough. Low noise treated instrumentation cables, such as Endevco 3090, generally will produce less than .02 pC/psi/ft.

These steps have been illustrated by calculations for a 2213C. Because of similarities in construction, the results computed for the 2213C will apply, except for low frequency pressure sensitivity, to all ENDEVCO®SEC accelerometers. That list includes Models 2211C, 2213C, 2215C, 2231C, 2232C, 2233, 2234, 2235C, 2242C, 2246, 2247, and 2248.

SUMMARY

Early testing in the field of acoustic response of piezoelectric accelerometers has relied on the use of a soft suspension to prevent the test unit from reacting to vibration of the chamber walls. Data so obtained were adequate to identify the few accelerometer designs which were too acoustically sensitive for environments of 5 or more years ago. Increasing severity of test environments now requires that an attempt be made to find out if acoustic outputs of most accelerometers are really as high as such testing indicates. The principle contribution of this paper to the attempt is development of a theory which shows:

(a) That the actual acoustic response of many piezoelectric accelerometers is about two orders of magnitude less than freely suspended acoustic testing would indicate. A substantial part of the actual response, in addition, arises from the interaction of the test unit and fixtures with the acoustic field, and large variations in output are possible at wavelengths approaching the transducer dimensions (typically about 10 kHz).

(b) That virtually all of the output of a freely suspended accelerometer can be accounted for by rigid body motions induced by the action of pressure gradients.

(c) That even a well designed fixture, intended to eliminate or compensate for these rigid body motions, may introduce an order of magnitude error at frequencies approaching 10 kHz.

Experimental data are available which support the fully developed theory in enough particulars to provide confidence in its correctness. Application of this theory to most piezoelectric accelerometers will show that acoustic sensitivity is not an important source of error at frequencies above a few thousand Hz, and rarely important at low frequencies.

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