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Instruments and Methods for Measuring Mechanical Impedance

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INSTRUMENTS AND METHODS FOR MEASURING MECHANICAL IMPEDANCE

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INTRODUCTION

Various methods have been employed for the experimental determination of the mechanical impedance of structures.

An indirect method [1] employs an electrodynamic vibration exciter equipped with both a driving coil and velocity sensing coil. The transfer electrical impedance of the coils is measured with known mechanical impedances attached to the exciter and with the structure of unknown impedance attached. The mechanical impedance of the structure is computed from equations relating it to the measured electrical impedances. Other indirect methods for measuring mechanical impedance include (a) measuring the terminal electrical impedance of a variable reluctance transducer under blocked and free conditions and (b) measuring the change in response of two identical reeds of known mechanical impedance when one is connected to the structure of unknown impedance. These methods and others are briefly discussed by Coleman[2].

A transient method also has been used for measuring mechanical impedance[3]. A transient force is applied and the resulting motion of the structure is measured with an accelerometer. The impedance is determined from "the ratio of the Fourier spectra of the force and response histories".

The most direct technique to determine mechanical impedance is to utilize force and motion measuring transducers. Recently a number of experimentists [2-10] have employed this technique on structures. Since this technique is quite widely accepted, the remainder of this paper is devoted to a calibration and evaluation of an impedance head and its use in measuring the impedance of a simple structure. A mechanical impedance head is a transducer with built-in force and motion measuring devices. Even though one type of impedance head is herein described in detail, the calibration procedures and discussion of accuracies are applicable to all transducers and their use in making mechanical impedance measurements.

DESCRIPTION OF IMPEDANCE HEAD

Mechanical impedance is defined as the ratio of the force exciting a structure to the resulting velocity. The motion transducer in the impedance head may be a displacement, velocity, or acceleration transducer. Plotting the apparent stiffness (ratio of force to displacement) or apparent weight (ratio of force to acceleration) on suitable graph paper provides directly the magnitude of the mechanical impedance when the excitation is sinusoidal. Sinusoidal excitation also makes it possible to utilize a variety of instruments for accurately measuring the phase angle of the impedance.

Most impedance measurements are made in the frequency range from 10 cps to 5,000 cps. In order to have wide application, the impedance head transducers should be flat (free of frequency distortion) and have zero phase angle between the force and motion transducers throughout this frequency range. If the phase angle is not zero, the data may be corrected if the phase shift is known and the excitation sinusoidal. The transducers must also be amplitude linear in order to measure a wide range of impedances. Mechanical impedance measurements usually are made at forces ranging from less than 1 pound up to about 100 pounds, and accelerations up to several hundred times the acceleration of gravity. Even very heavy structures can be measured in the low force ranges since the frequencies of primary interest are those above which the structure acts as a rigid body. Significant motions result from small forces near resonances, i.e. regions of low impedance. The impedance head must also measure motions corresponding to accelerations much less than 1 g (386in. /sec^2) in order to measure accurately high impedances at

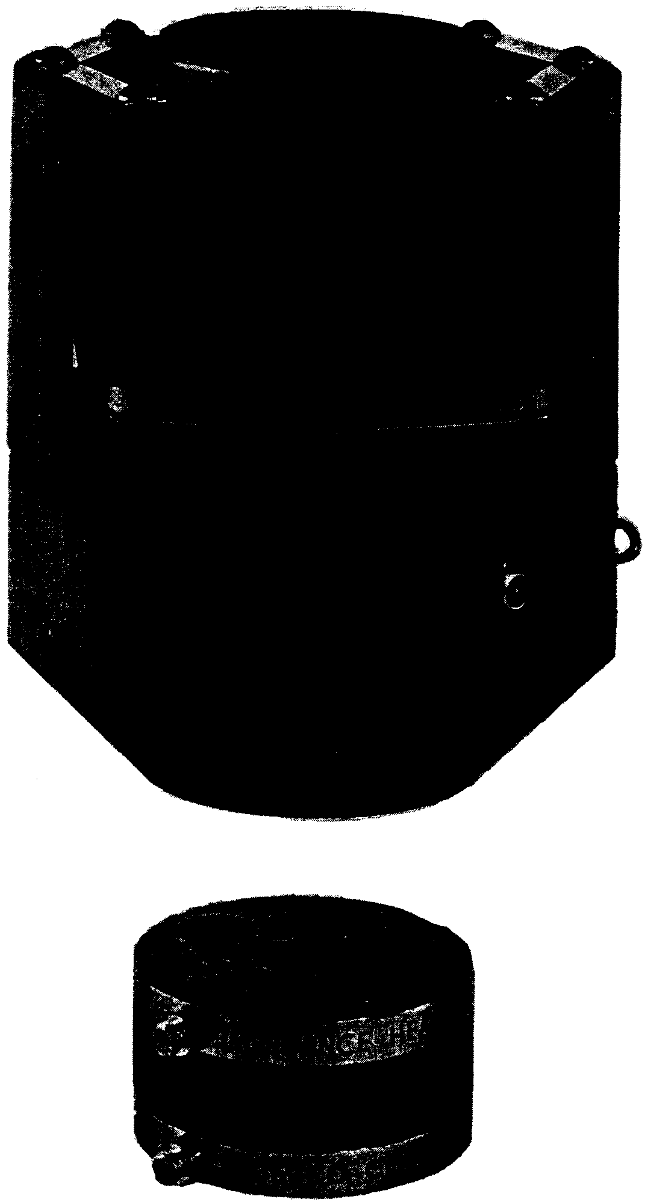

Figure 1. Electrodynamic force generator and impedance head used for measuring the impedance of structures.

anti-resonances in the structure. For this reason, accelerometers are preferred for making impedance measurements at high frequencies since the outputs from velocity and displacement pickups are quite small for small amplitude motions at frequencies much above 1000 cps.

Another requirement for accurately measuring large impedances is that the motion transducer should not respond to force excitation particularly when the acceleration tends to zero. If false outputs result from the motion transducer due to dynamic stresses applied to the case of the impedance head, the indicated impedance would be less than the actual impedance. The impedance head should be designed so this interaction does not occur.

On the other hand, the interaction of the force transducer responding to applied accelerations normally occurs in an impedance head. This output from the force transducer equals the mass of the end of the impedance head plus any externally attached mass times their acceleration. This characteristic is used to dynamically calibrate the force transducer in the impedance head.

The mass of the impedance head should be small and its stiffness large so that the effect of the impedance head on the response of the structure is not significant. If the head has these characteristics, the measured data on many structures need not be corrected for the head mass and stiffness.

One type of impedance head and an accompanying force generator are illustrated in Figure 1. The impedance head is 1 inch high and nearly 2 inches in diameter. It has three piezoelectric force pickups and three piezoelectric accelerometers built into the head. A bolt is attached to the force generator and passed through the hole in the center of the head to rigidly connect the head and outer casing of the impedance head to a structure. The force generator consists of an electrodynamic vibration exciter with its driving coil rigidly connected to the outer casing. The permanent magnet of the exciter which serves as a reaction mass is flexibly attached to the casing by thin leaf springs. It is rated at 1 pound driving force which is sufficient for testing many structures.

CALIBRATION

Sensitivity The sensitivities of the force pickup

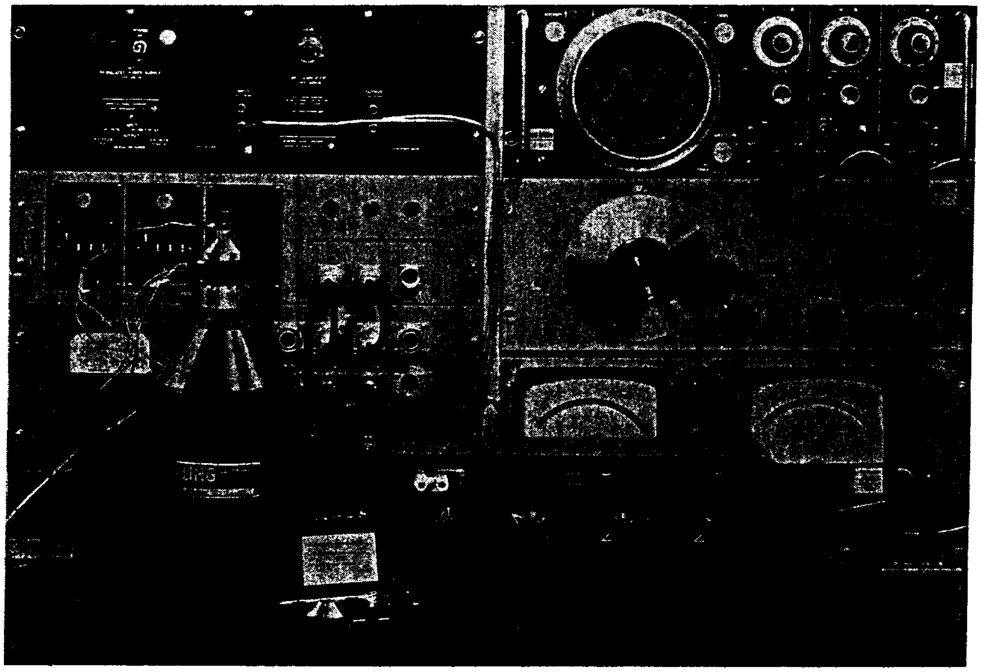

Figure 2. Setup used to calibrate the force pickup and accelerometer in the impedance head.

and accelerometer parts of the impedance head were performed on an electrodynamic vibration exciter using the test setup illustrated in Figure 2. The sensitivity of the force pickup was determined by measuring its output with two different weights attached to the impedance head. The ratio of the outputs from the force pickup to standard accelerometer mounted on top of the impedance head was measured. The measurement was repeated with a 2 pound weight inserted between the standard pickup and tapered adapter block on top of the impedance head. The force sensitivity was the change in force pickup output divided by 2 pounds and the applied acceleration. From this force sensitivity determination performed at one frequency, the total weight acting on the force pickup including the effective portion of the mounting bolt through the head and the effective end weight of the head was determined. Knowing the total weight the frequency response was

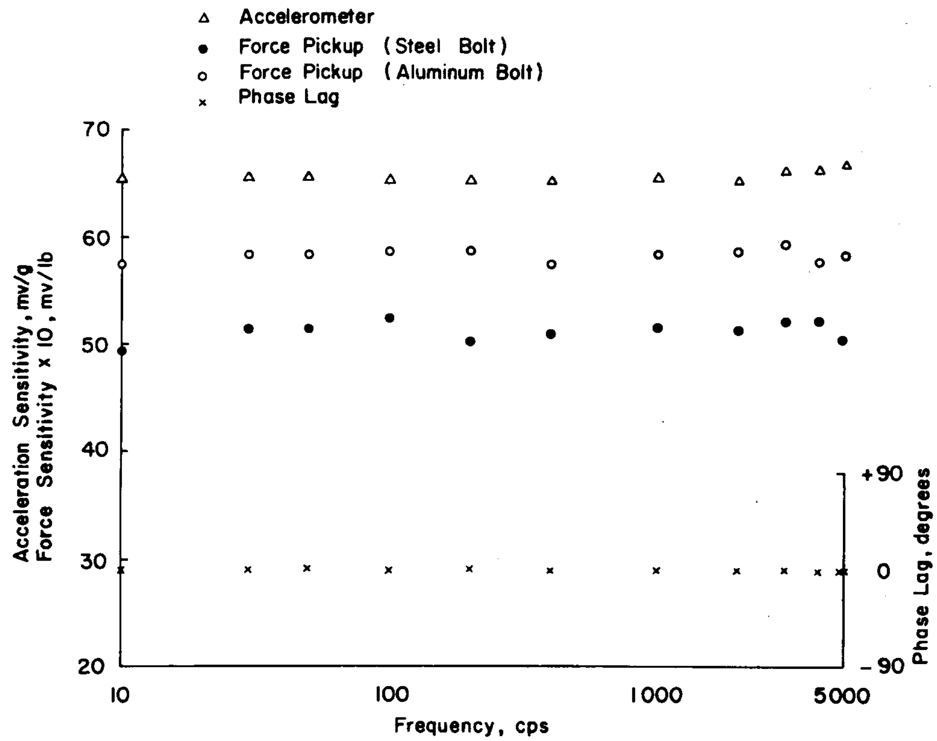

Figure 3. Calibration results on an impedance head. The force pickup and accelerometer were calibrated by comparison with a standard accelerometer. The phase angle measurements were made with a dual-beam oscilloscope.

determined by measuring the force pickup to standard accelerometer outputs from 10 cps to 5,000 cps without the 2 pound weight attached. This part of the calibration was repeated measuring the ratio of the output of the accelerometer in the head to the standard accelerometer output to determine the accelerometer sensitivity over the same frequency range. The 2 pound weight was not used at the higher frequencies in order to avoid relative motion between the standard accelerometer and impedance head. A voltage ratio circuit [11] was used to reduce calibration errors.

The results of the calibration are shown in Figure 3. The sensitivity of the accelerometer in the impedance head is nearly constant throughout the frequency range, increasing slightly near 5,000 cps. Also, the sensitivities of the force pickup are nearly constant in the same frequency range. The sensitivity for use with an aluminum 1/2-inch diameter bolt is 5.8 mv/lb. The sensitivity is reduced to 5.1 mv/lb when a 1/2-inch diameter steel bolt is used. This change in sensitivity occurs since a small portion of the total force is applied to the bolt while the rest of the force is applied to the head. Of course, the force sensitivity is the voltage output divided by the total force. The portion of the total force applied to the bolt is the ratio of the bolt stiffness to head stiffness. The impedance head stiffness computed from the modulus of elasticity and geometric dimensions of the various parts is 2.7×10^7 lb/in. Therefore, the change in sensitivity resulting from using different size bolts can be computed. The computed change in sensitivity for the steel and aluminum 1/2-inch diameter bolts is approximately 12 per cent which agrees closely with the results shown in Figure 3. When no bolt is used, the increase in sensitivity is 7 per cent compared to the value obtained with the aluminum bolt.

The phase angle between the force and acceleration outputs is zero degrees throughout the frequency range. This indicates that the acceleration motion and force applied to a structure are faithfully reproduced in the output signals from the head and no correction to the measured phase angle of the impedance need be made.

The effective end weight of the impedance head was measured by repeating the above described force calibration with several different weights attached. The output from the force pickup corresponding to 1 g acceleration was measured for each weight applied externally to the impedance head. These data were plotted and the points fitted with a least squares line. The results for two impedance heads are shown in Figure 4. The intercepts of the lines with the abscissa indicates the effective end weight of the impedance head is about 0.3 pound.

Amplitude Linearity The amplitude linearities of the impedance head were verified on electrodynamic vibration exciters equipped with resonant beams and on a shock motion calibrator.

Calibrations up to 112 pounds and 100 g were made on vibration exciters. With a small weight on top of the impedance head, sufficiently high accelerations were achieved by mounting it at the center of a free-free beam. In order to achieve sufficiently high forces, it was necessary to mount the head on the end of the beam. A

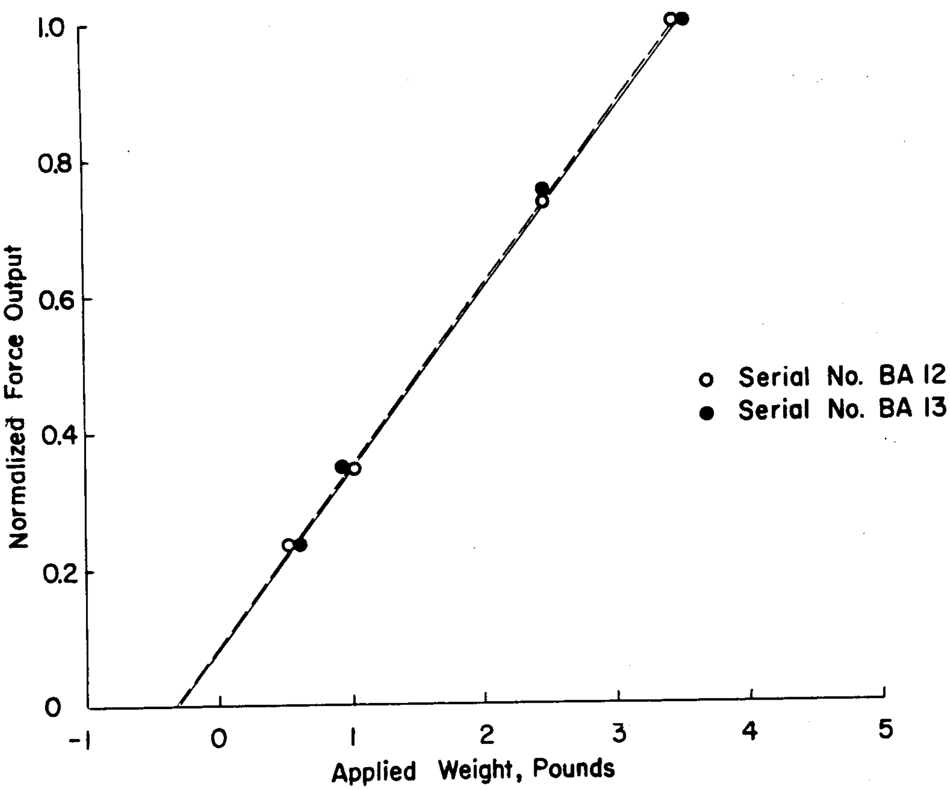

Figure 4. Normalized force output measured with different applied weights. The impedance head apparent end-weight is indicated by the abscissa intercept, 0.3 pound.

standard accelerometer was mounted on top of the head and the calibration was made using the same method used for the above sensitivity calibrations.

Accelerations up to 502 g were applied on shock motion calibrators similar to that described in reference [12]. The impedance head and standard accelerometer were attached to an anvil. Two-inch diameter and four-inch diameter hardened steel balls were dropped on the anvil. Rubber paddings were used on the impact surface to obtain the desired accelerations. The amplitude linearity was determined by measuring the ratio of the standard accelerometer to impedance head accelerometer outputs which were photographed on the screen of an oscilloscope. The results are given in Table 1 and Figure 5. The deviations from amplitude linearity are less than the measurement

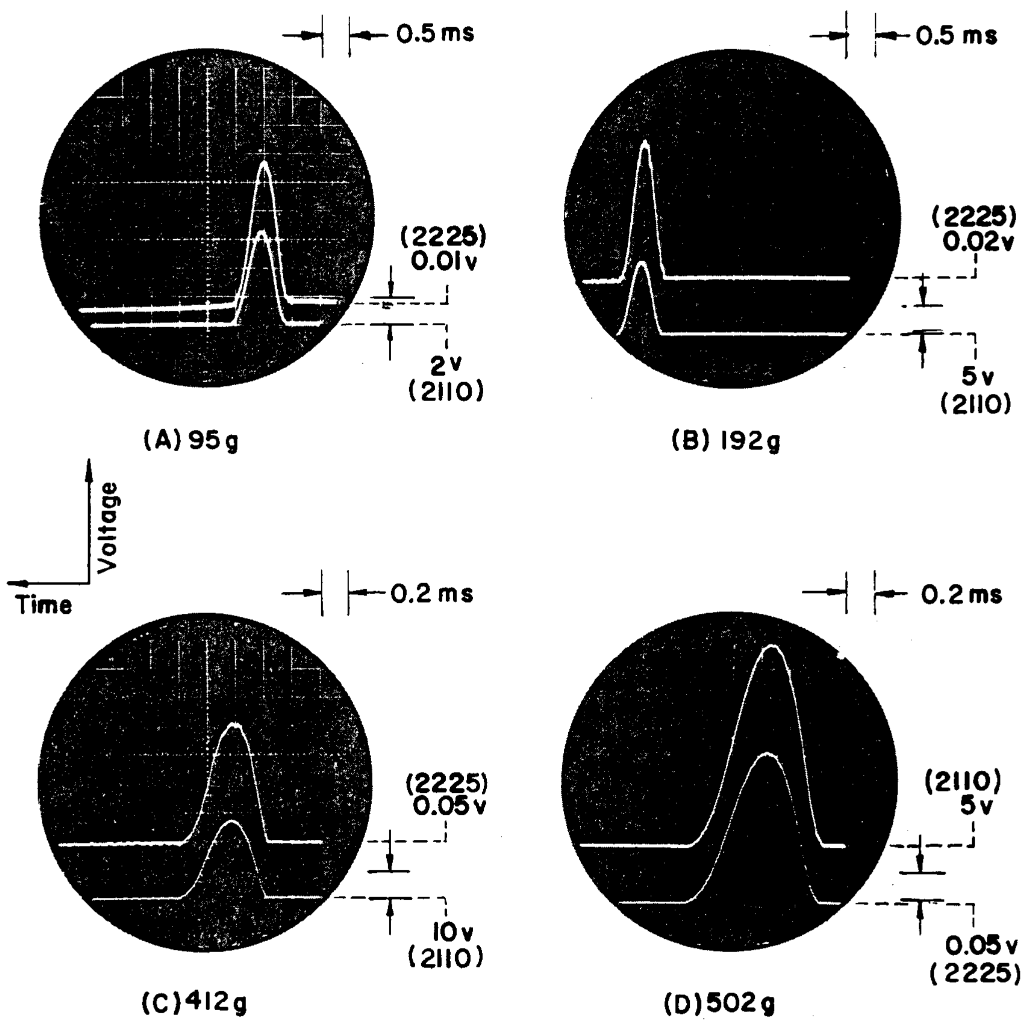

Figure 5. Comparison calibrations performed on a shock machine to verify the amplitude linearity of the accelerometer in an impedance head. The standard accelerometer (2225) and impedance head (2110) were mounted on an anvil in a drop-ball shock machine and the ratio of the outputs compared.

Table 1. Amplitude linearity of Endevco impedance head Model 2110

Force Gage		Accelerometer	
Applied force	Deviation	Applied acceleration	Deviation
lb	per cent	g	per cent
0.03	+1.0	0.03	0.0
0.1	+0.5	0.1	0.0
1	0.0	1	0.0
17	-0.5	10	0.0
28	0.0	50	0.0
56	0.0	100	+0.4
70	0.0	95	0.0
84	0.0	192	+0.1
112	+0.5	412	+0.9
		502	0.0

errors. The Model 2225 standard accelerometer, Figure 5, was previously calibrated throughout its useful acceleration range by the absolute method described in reference [12].

Resonance Frequency Resonance frequency measurements were also made on the shock machine. Less padding was used on the anvil to reduce the pulse duration in order to produce resonance frequency excitation in the impedance head. The results are shown in Figure 6. The oscillogram, Figure 6A, indicates the resonance frequency of the accelerometer part of the impedance head is about 28,000 cps. The theoretical response for an ideal pickup, equation 10 in reference [13], with a 28,000 cps resonance frequency is in close agreement with the slight rise near 5,000 cps noted in the sinusoidal calibration, Figure 3, of the impedance head. The beating present in Figure 6A apparently is due to the individual accelerometers in the head having slightly different resonance frequencies. The accelerometer part of the impedance head consists of three individual accelerometers connected in parallel with a single output.

The response was measured also with the force pickup in the impedance head subjected to shock excitation. Measurements were made on two different structures. The first structure consisted of 4 parts including the impedance head for a total length of about 4 inches. The second structure had 5 parts for a total length of seven inches.

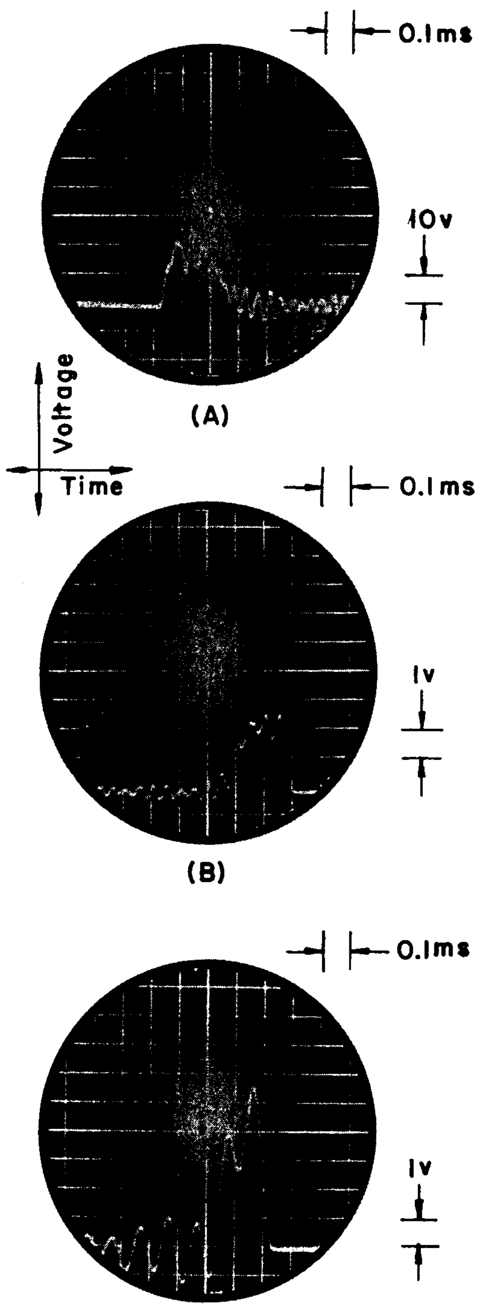

Figure 6. Resonance response of an impedance head. (A) Resonance frequency of the accelerometer part of the impedance head is 28,000 cps. (B) and (C) Force response with the impedance head mounted in two structures resonate at 20,400 cps and 10,300 cps, respectively.

The corresponding responses from the force pickup indicates a structural resonance of 20,400 cps and 10,300 cps, Figures 6B and C, respectively. These frequencies correspond to the resonance frequencies for the above two structures. This indicates that the impedance head is capable of making measurements at very high frequencies. Considering the mass-stiffness characteristics of the impedance head only, computations would indicate its resonance frequencies would occur at frequencies much higher than those present in Figures 6B and C.

Other Characteristics In addition to the above calibrations, tests were performed to establish the other characteristics of the impedance head.

The force sensitivity of the accelerometer part of the impedance head was determined on a hydraulic testing machine. The applied load was released instantaneously with a quick acting valve. An

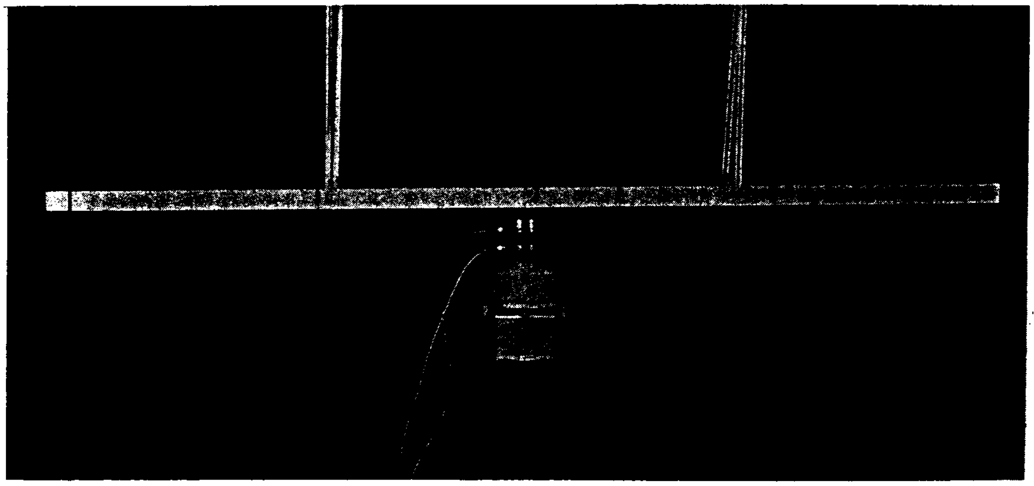

Figure 7. Mechanical impedance head and force generator connected to a free-free beam. The beam is suspended from overhead by rubber bands.

oscilloscope was used to measure the output from the accelerometer. No significant output from the accelerometer was measured with an applied load of 5,000 lb which was the capacity of the testing machine. This indicated the response of the accelerometer is of the order of 0.0001 g/lb or less which corresponded to the resolution obtainable with the test equipment. Since this interaction in the impedance head is so small, it is capable of measuring very large impedances accurately.

The other characteristics including transverse sensitivity of the accelerometer, capacitances, internal resistances, etc. are typical to that of other piezoelectric transducers. Both the force pickup and accelerometer are insulated from the case of the impedance head in order to eliminate electrical noise in auxiliary electronic instruments.

IMPEDANCE OF A BEAM

In order to illustrate typical ranges of impedance and frequency, measurements were made on an uniform beam. The test setup is illustrated in Figure 7. The force generator and impedance head were connected rigidly to a

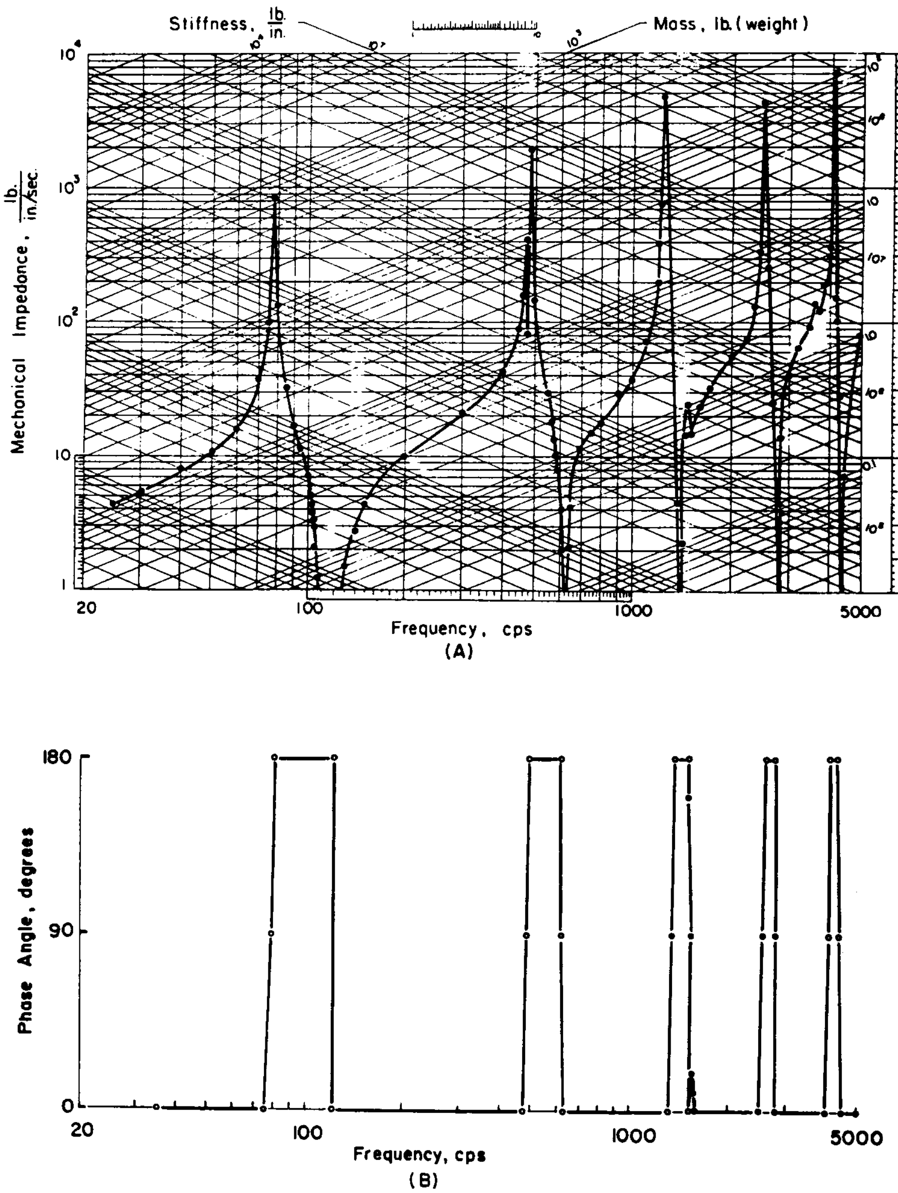

Figure 8. Mechanical impedance of an aluminum beam measured with an impedance head: (A) The magnitude of the impedance, apparent weight, or apparent stiffness is indicated by the appropriate scales. (B) This plot is the phase angle of the apparent weight.

Table 2. Resonance frequencies of a beam measured with an impedance head

Measured resonance frequency	Theoretical resonance frequency	Normal mode	
		Number	Type
cps	cps		
78	74	1st	Fixed-free
121	117	1st	Free-free
482	460	2nd	Fixed-free
619	631	3rd	Free-free
1340	1289	3rd	Fixed-free
1513	1560	5th	Free-free
2560	2526	4th	Fixed-free
2790	2900	7th	Free-free
4080	4176	5th	Fixed-free
4400	4652	9th	Free-free

3/4in. x 3in. x 36in. aluminum beam (2024-T4) with a 1/2-inch diameter steel bolt. The bolt passed through a steel tapered adapter block provided with a 1/2-inch clearance hole. The contact area of the block with the beam is an annulus with an area of 0.23 in.^{2}. The force and acceleration outputs from the impedance head were connected to a pair of voltage amplifiers and directly to voltmeters and a dual beam oscilloscope. No filtering circuits were used. The power amplifier used with the force generator was operated at sufficiently low levels so no distortion was present except through very narrow frequency ranges at resonance where the force or acceleration tends to zero. The magnitude of the impedance was computed from the measured voltage outputs times the force pickup and accelerometer sensitivities determined from the above calibrations. No corrections were made to the data for the apparent weight of the impedance head, bolt, or tapered adapter block. The phase angle of the apparent weight was measured directly on a dual-beam oscilloscope and by Lissajous figures.

The results are given in Figure 8. The experimentally measured resonances and the theoretically computed resonances for the beam are given in Table 2. The cantilever beam resonance at 78 cps was characterized by a single nodal point at the center of the beam and large displacements at both beam ends. The free-free beam frequencies excited were odd numbered modes characterized by an anti-nodal point at the center of the beam. It was difficult to excite the even-mode frequencies due to the effect of the bolt and adapter block when the center of the beam tends to form an inflection point. It is interesting to note that an attempt to correct for the apparent mass of the impedance head, bolt, and adapter block would not alter the results at or near the anti-resonances (regions of high impedances). Also, an attempt to correct for this apparent mass at resonances (regions of low impedances) would produce only small differences between the measured results and theoretical resonances. In the case of this beam, good results were obtained without correcting for the impedance head and fixture apparent weight.

CONCLUSIONS

Although several different methods have been used to measure mechanical impedance, the direct measurement method with force and motion transducers is becoming attractive. Impedance heads are available for making measurements over wide frequency and impedance ranges. Accurate impedance measurements are easily made with small heads that are sufficiently stiff. Measurements made on an aluminum beam indicate good accuracy is maintained without correcting for the effective end-mass of the impedance head.

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